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GENERAL OPTICAL DIFFRACTION-STRAIN RELATIONS

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ABSTRACT

General diffraction-strain equations are developed for the optical diffraction grating strain gages now coming into use in connection with experimental studies of wave propagation in solids. Equations are derived for the following cases: superimposed gratings crossed at any angle and subjected to arbitrary surface strains; orthogonal gratings subjected to arbitrary rotations; and orthogonal gratings aligned with principal strain directions and subjected to arbitrary rotations during strain.

The inherent strain resolution capability of an optical diffraction system is discussed and the differences between this technique for strain measurement and that of x-ray stress analysis are described.

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LIST OF SYMBOLS

$\bar{a}_o$ - Lattice vector of undeformed lattice

$\bar{a}^*_o$ - Reciprocal lattice vector of undeformed lattice

$\bar{a}$ - Lattice vector of deformed lattice

$\bar{a}^*$ - Reciprocal lattice vector of deformed lattice

A - Rotation matrix

a, b, c - Rotation matrices

a´, b´, c´ - Rotation matrices

$\bar{b}_o$ - Lattice vector of undeformed lattice

$\bar{b}^*_o$ - Reciprocal lattice vector of undeformed lattice

$\bar{b}$ - Lattice vector of deformed lattice

$\bar{b}^*$ - Reciprocal lattice vector of deformed lattice

$\bar{c}_o$ - Normal vector to undeformed lattice

$\bar{c}^*_o$ - Reciprocal normal vector to undeformed lattice

$\bar{c}$ - Normal vector to deformed lattice

$\bar{c}^*$ - Reciprocal normal vector to deformed lattice

$\hat{d}^{(hk)}$ - Unit diffraction vector before deformation

$\hat{d}^{o(hk)}$ - Unit diffraction vector during deformation

D - Distance from grating to observation screen

$\bar{D}^{(hk)}$ - Diffraction vector before deformation

$\bar{D}^{o(hk)}$ - Diffraction vector during deformation

$\hat{e}_i$ - Unit vector of the undeformed lattice

$\hat{e}_i'$ - Unit vectors

$\hat{e}_i''$ - Unit vectors

h - Order number

i - Angle of incidence for a line grating

$\hat{i}_o$ - Unit vector in the direction of the incident light

k - Order number

L_g - Length of diffraction grating strain gage

m - Reciprocal line spacing of line grating

n - Order number

$\hat{N}_o$ - Unit normal to undeformed lattice

N - Total number of lines of a line grating

$P_a^{\,o}$ - Perpendicular line spacing of lines parallel to $\bar{a}_o$

$P_b^{\,o}$ - Perpendicular line spacing of lines parallel to $\bar{b}_o$

P_a - Perpendicular line spacing of lines parallel to $\bar{a}$

P_b - Perpendicular line spacing of lines parallel to $\bar{b}$

R_ε - Strain resolving power of an optical diffraction grating

R_λ - Chromatic resolving power of an optical diffraction grating

$\hat{u}_i$ - Unit vectors of the deformed lattice

α_i - Coefficients of $\hat{d}_o^{(hk)}$ with respect to $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$.

α_o, β_o, γ_o - Coefficients of $\hat{d}_o^{(hk)}$ with respect to $\bar{a}_o^*$, $\bar{b}_o^*$ and $\bar{c}_o^*$

β_j - Coefficients of $\hat{d}^{(hk)}$ with respect to $\hat{u}_1$, $\hat{u}_2$ and u_3

β_j' - Coefficients of $\hat{d}^{(hk)}$ with respect to $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$

Γ_{ab} - Shear strain of lattice

ε_a - Extension of lattice vector $\bar{a}_o$

ε_b - Extension of lattice vector $\bar{b}_o$

ε - Strain of a line grating

$|\varepsilon|_{min}$ - Smallest strain resolvable by a line grating

θ_n - Diffraction angle of n^{th} diffraction order

$\Theta_o^{(hk)}$ - Angle between $\hat{d}_o^{(hk)}$ and $\hat{d}_o^{(oo)}$ before deformation

$\Theta^{(hk)}$ - Angle between $\hat{d}^{(hk)}$ and $\hat{d}_o^{(oo)}$ during deformation

λ - Wave length of incident radiation

ϕ, θ, ψ - Euler angles

ω - Axis of tilt of the grating plane

GENERAL OPTICAL DIFFRACTION STRAIN RELATIONS

I. INTRODUCTION

The new methods for analysis of mechanical strains by optical diffraction techniques cannot be discussed without reference to the conceptually related method of strain analysis by x-ray diffraction. One might be tempted to look to this older method for guidance in developing suitable equations describing diffraction by visible light. In fact, there is very little that can be adopted from the established x-ray methods beyond the application of the familiar Laue equations and the liberal use of the reciprocal lattice concept. The need for the specific equations newly developed in the following report arises from the desire to measure strains in dynamic loading which are accompanied by surface rotations. These rotations can take the form of shear strains or of actual rigid body rotation. The separation of strains and rigid body rotation is imperative if the measurements are to be meaningful. Exploring the extent to which this can be accomplished is one of the main goals of this report.

Using the x-ray method (1) as a point of departure, it should be recognized that the underlying principles of the optical method are the same. The differences, however, are significant: The lattice seen by x-rays is a 3-dimensional array of atoms and requires that 3 Laue conditions be satisfied simultaneously, placing rather stringent restrictions on the phenomenon. An attempt to measure a strain in a single crystal with the use of monochromatic x-rays would lead to a total loss

of the diffraction spot. In the manmade optical grating formed into the surface of a specimen at least one of the three Laue conditions is relaxed. Consequently, a complete image of a reciprocal lattice section always remains visible when the monochromatic light of a laser is used. The x-ray method requires polycrystaline materials to satisfy the 3 Laue equations with sufficient frequency to produce useful information and the results are more conveniently expressed in the language of Bragg's law. Then the two methods have very little in common. It might be noted that the only case of an x-ray technique related to the present work is found in the back-reflection method of orientation determination for single crystals in which the Bragg reflections from the faces of a crystal zone share some rotation properties with the diffractions from one or two dimensional optical gratings. Comparing the two techniques in their applications, the following peculiarities stand out. Both methods measure strains in the surface of a solid; the x-ray method can also detect a strain perpendicular to the surface but, because of high absorption, only in a thin surface layer. The x-ray method has some serious limitations. Thus, (a) the sample must be crystalline so that most plastics are automatically excluded, (b) only elastic strains can be detected, and (c) even with flash x-ray sources, high speed applications in the microsecond range are not feasible yet. The x-ray method does have an inherent strain resolution that is much higher than that of the optical grating. It follows that the optical grating is most suitable for high speed measurement of plastic strains and is limited only by strains high enough to impair the structure of the grating to an extent that it ceases to function as an ordered diffraction source.

Previous workers (2, 3, 4, 5, 6, 7) utilizing the diffraction grating strain gage have derived their diffraction-strain relations from the equation

$$\sin\theta_n - \sin i = n\lambda m \tag{1}$$

where θ_n = the diffraction angle of the n^{th} direction of constructive interference measured from the normal to the grating

i = the angle of incidence of the light measured from the normal

n = the order number

λ = the wave length of the radiation

m = the reciprocal line spacing of the grating.

This equation applies only to a line grating where the direction of the incident illumination is required to be perpendicular to the lines of the grating but is not necessarily perpendicular to the grating plane. When this is the case, the diffraction orders as seen on a plane screen will lie along a straight line. The spacing of the orders along such a line is easily determined from equation (1).

If the direction of the incident light is not perpendicular to the grating lines, the locus of the diffraction spots on the observation screen will be curved. When this is the case equation (1) is inapplicable. It then is necessary to use a more general diffraction equation valid for an arbitrary direction of the incident light.

The use of equation (1) involves some basic assumptions about the behavior of the grating during deformation. If diffraction-strain relations derived from equation (1) are to be absolutely correct then the diffraction spots must remain on a straight line during deformation of the grating. For this reason those rigid rotations of the grating which would result in a

curved pattern must be excluded. A tilt of the grating plane about any line which is parallel to the lines of the grating would not alter the straight line character of the diffraction pattern. However, all other rotations must be either zero or small enough to produce a negligible effect on the diffraction pattern. When the grating lines are not parallel to a principal strain direction, large shear strains will rotate the grating lines. This rotation could cause a curved locus of diffraction spots not adequately described by the equation. To avoid this behavior the lines of the grating are usually ruled in anticipated directions of principal surface strain. Thus a diffraction-strain theory based on equation (1) is limited to a grating whose lines are parallel to a principal strain direction and whose rotation axis is that same principal direction. When other rotations or shear strains are present the use of equation (1) could lead to serious inaccuracy.

A disadvantage of the line grating strain gage is that it measures only one strain component. A crossed grating provides sufficient information to determine the entire state of surface strain. Unfortunately, the existing diffraction-strain equations can be used with crossed gratings in only very limited cases.

There are several reasons for developing general diffraction-strain relations. These equations can be used to determine the accuracy of existing ones. They apply to crossed gratings and exhibit the effects of all rigid rotations. In some cases shear strain effects can also be incorporated into the equations.

The purpose of this report is to present some general diffraction-strain equations which apply to both line and crossed gratings. Equations

derived first are for a crossed grating but with no rotations. These are followed by a description of rigid rotations. Then general diffraction-strain equations that include rotation effects are obtained. A discussion of the resolution capabilities of the grating strain gage is also provided.

II. THE RESPONSE OF A CROSSED GRATING TO ARBITRARY SURFACE STRAINS IN THE ABSENCE OF SURFACE TILT

A crossed grating has, as the name implies, two sets of parallel lines. The resulting grid can be approximated by a two dimensional doubly periodic lattice. The points of intersection of the lattice lines can be considered as secondary sources of light in the Huygens' sense. A direction of constructive interference occurs when the light from any four adjacent sources differs in phase by integral numbers of wave lengths. The geometry of the undeformed lattice is determined by the lattice vectors $\bar{a}_o$ and $\bar{b}_o$ as shown in Figure 1. Vector $\bar{c}_o$ is normal to the lattice plane and has the physical dimension of length as do $\bar{a}_o$ and $\bar{b}_o$. The direction of the incident light is given by vector $\hat{i}_o$. A typical direction of constructive interference is shown as vector $\hat{d}_o$. These vectors must satisfy the equations

$$(\hat{d}_o^{(hk)} - \hat{i}_o) \cdot \bar{a}_o = h\lambda \qquad , h = o, \pm 1, \pm 2, \ldots, m \tag{2}$$

$$(\hat{d}_o^{(hk)} - \hat{i}_o) \cdot \bar{b}_o = k\lambda \qquad , k = o, \pm 1, \pm 2, \ldots, n \tag{3}$$

$$|d_o| = 1 \tag{4}$$

and $$|i_o| = 1 \tag{5}$$

The subscript o refers to the undeformed configuration. These equations are the well known Laue equations of x-ray diffraction theory see, for example, Azaroff (8). The directions of constructive interference are

completely determined if the lattice vectors, the direction of incidence and the wave length are specified.

Equations (2) through (5) would also apply to the deformed lattice. Hence to determine diffraction-strain equations one must solve these equations for both the undeformed and the deformed lattice vectors.

A solution to equations (2) through (5) is facilitated by introducing reciprocal lattice vectors $\bar{a}_o*$, $\bar{b}_o*$ and $\bar{c}_o*$. These vectors are determined from the lattice vectors by the relations

$$\bar{a}_o* = \frac{\bar{b}_o \times \bar{c}_o}{\bar{a}_o \cdot (\bar{b}_o \times \bar{c}_o)} \tag{6}$$

$$\bar{b}_o* = \frac{\bar{c}_o \times \bar{a}_o}{\bar{a}_o \cdot (\bar{b}_o \times \bar{c}_o)} \tag{7}$$

$$\bar{c}_o* = \frac{\bar{a}_o \times \bar{b}_o}{\bar{a}_o \cdot (\bar{b}_o \times \bar{c}_o)} \quad . \tag{8}$$

Reciprocal vectors $(\bar{a}_o*, \bar{b}_o*)$ will be shown to be directly related to the observed diffraction pattern. The reciprocal vectors satisfy the following conditions

$$\left.\begin{array}{lll} \bar{a}_o*\cdot\bar{a}_o = 1 & \bar{a}_o*\cdot\bar{b}_o = 0 & \bar{a}_o*\cdot\bar{c}_o = 0 \\ \bar{b}_o*\cdot\bar{a}_o = 0 & \bar{b}_o*\cdot\bar{b}_o = 1 & \bar{b}_o*\cdot\bar{c}_o = 0 \\ \bar{c}_o*\cdot\bar{a}_o = 0 & \bar{c}_o*\cdot\bar{b}_o = 0 & \bar{c}_o*\cdot\bar{c}_o = 1 \end{array}\right\} \tag{9}$$

A solution is obtained by finding α_o, β_o, and γ_o for which

$$\hat{d}^{(hk)} = \alpha_o a_o* + \beta_o b_o* + \gamma_o c_o* \quad . \tag{10}$$

Making use of equation (10) in equations (2) and (3) gives

$$\alpha_o = \hat{i}_o \cdot \bar{a}_o + h\lambda \tag{11}$$

and

$$\beta_o = \hat{i}_o \cdot \bar{b}_o + k\lambda \ . \tag{12}$$

If equations (10) and (4) are combined there results

$$1 = \alpha_o{}^2|\bar{a}_o*|^2 + \beta_o{}^2|\bar{b}_o*|^2 + \gamma_o{}^2|\bar{c}_o*|^2 + 2\alpha_o\beta_o\bar{a}_o*\cdot\bar{b}_o* \ . \tag{13}$$

Equation (13) yields

$$|\bar{c}_o*|\gamma_o = + \sqrt{1 - \alpha_o{}^2|\bar{a}_o*|^2 - \beta_o{}^2|\bar{b}_o*|^2 - 2\alpha_o\beta_o\bar{a}_o*\cdot\bar{b}_o*} \tag{14}$$

where the plus sign is chosen for a reflection lattice. One can easily show from equation (6), for $\bar{c}_o$ normal to $\bar{a}_o$ and $\bar{b}_o$, that

$$|\bar{a}_o*| = \frac{1}{|\bar{a}_o|\sin(\bar{a}_o,\bar{b}_o)} \tag{15}$$

where $\angle(\bar{a}_o, \bar{b}_o)$ is the angle between the lattice vectors. Likewise, it follows from equation (7) that

$$|\bar{b}_o*| = \frac{1}{|\bar{b}_o|\sin(\bar{a}_o,\bar{b}_o)} \ . \tag{16}$$

From Figure 2 one can see that

$$P_b{}^o = |\bar{a}_o|\sin(\bar{a}_o,\bar{b}_o) \tag{17}$$

and

$$P_a{}^o = |\bar{b}_o|\sin(\bar{a}_o,\bar{b}_o) \tag{18}$$

where $P_b{}^o$ and $P_a{}^o$ are perpendicular distances between lattice lines. The reciprocal vectors to $\bar{a}_o$ and $\bar{b}_o$ are also shown in Figure 2. Reciprocal vectors $\bar{a}_o*$ and $\bar{b}_o*$ define a new lattice which is reciprocal to the lattice of $\bar{a}_o$ and $\bar{b}_o$. From equations (15) through (18) it follows that the magnitudes of the reciprocal vectors are

$$\left.\begin{aligned} |\bar{a}_o^*| &= \frac{1}{P_b^{\,o}}\,, \\ \text{and}\quad |\bar{b}_o^*| &= \frac{1}{P_a^{\,o}}\,. \end{aligned}\right\} \quad (19)$$

Equations (19) show the reciprocal character of the two lattices. The angles between the lattice vectors of each lattice are also related. From equations (6) and (7) the scalar product of the reciprocal lattice vectors $\bar{a}_o^*$ and $\bar{b}_o^*$ is

$$\bar{a}_o^* \cdot \bar{b}_o^* = -\frac{\cos(\bar{a}_o,\bar{b}_o)}{P_a^{\,o}\,P_b^{\,o}} \qquad (20)$$

where equations (19) have been used in the result. However, from the definition of the scalar product and equations (19)

$$\bar{a}_o^* . \bar{b}_o^* = |\bar{a}_o^*|\,|\bar{b}_o^*| \cos(\bar{a}_o^*,\bar{b}_o^*) = \frac{\cos(\bar{a}_o^*,\bar{b}_o^*)}{P_a^{\,o}\,P_b^{\,o}} \qquad (21)$$

where $\angle(\bar{a}_o^*,\bar{b}_o^*)$ is the angle between the reciprocal lattice vectors. Combining equations (20) and (21) yields

$$\cos(\bar{a}_o^*,\bar{b}_o^*) = -\cos(\bar{a}_o,\bar{b}_o). \qquad (22)$$

The solution to equation (22) that is pertinent to the physical case is

$$\angle(\bar{a}_o^*,\bar{b}_o^*) = \pi - \angle(\bar{a}_o,\bar{b}_o) \qquad (23)$$

Equation (23) relates the angle between the real lattice vectors to the angle between the reciprocal lattice vectors. Together equations (19) and (23) determine the geometry of the reciprocal lattice in terms of the real lattice. It will be shown later that the reciprocal lattice is directly related to the observed diffraction pattern.

The solution for $\hat{d}_o^{(hk)}$ is found by combining equations (11), (12), and (14) with equation (10) to obtain

$$\hat{d}_o^{(hk)} = (\hat{i}_o \cdot \bar{a}_o + h\lambda)\bar{a}_o^* + (\hat{i}_o \cdot \bar{b}_o + k\lambda)\bar{b}_o^* + \sqrt{1 - (\hat{i}_o \cdot \bar{a}_o + h\lambda)^2 |\bar{a}_o^*|^2 - (\hat{i}_o \cdot \bar{b}_o + k\lambda)^2 |\bar{b}_o^*|^2 - 2(\hat{i}_o \cdot \bar{a}_o + h\lambda)(\hat{i}_o \cdot \bar{b}_o + k\lambda) \times \bar{a}_o^* \cdot \bar{b}_o^*} \; \frac{\bar{c}_o^*}{|\bar{c}_o^*|} \tag{24}$$

Equation (24) gives the various directions of constructive interference in terms of lattice vectors and the vector of incidence. This equation is used to locate the orders on an observation screen or the film plane of a camera. Consider such an observation screen as shown in Figure 3. For convenience the screen is placed parallel to the lattice and the direction of incidence is chosen normal to the lattice. The distance between the screen and the lattice is D.

For normal incidence

$$\hat{i}_o \cdot \bar{a}_o = 0 \tag{25}$$

and

$$\hat{i}_o \cdot \bar{b}_o = 0 \tag{26}$$

Using equations (25) and (26) in equation (24) gives

$$\hat{d}_o^{(hk)} = h\lambda\bar{a}_o^* + k\lambda\bar{b}_o^* + \sqrt{1 - h^2\lambda^2|\bar{a}_o^*|^2 - k^2\lambda^2|\bar{b}_o^*|^2 - 2(h\lambda)(k\lambda)\,\bar{a}_o^* \cdot \bar{b}_o^*}\;\hat{N}_o \tag{27}$$

where

$$\hat{N}_o = \frac{\bar{c}_o^*}{|\bar{c}_o^*|} \tag{28}$$

is a unit normal vector of the lattice. For normal incidence the direction of zero phase difference as determined from equation (27) is

$$\hat{d}_o^{(oo)} = \hat{N}_o \quad . \tag{29}$$

This direction which is usually called the zero order of diffraction always corresponds to the direction of specular reflection of the incident light. The diffraction spot on the observation screen corresponding to

the zero order is located by vector

$$\bar{D}_o^{(oo)} = D\hat{d}_o^{(oo)} = D\hat{N}_o \quad . \tag{30}$$

The diffraction spot of order (h,k) is likewise located by vector

$$\bar{D}_o^{(hk)} = |\bar{D}_o^{(hk)}| \; \hat{d}_o^{(hk)} \tag{31}$$

where $|\bar{D}_o^{(hk)}|$ is to be determined.

Vector
$$\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)} = |\bar{D}_o^{(hk)}|\hat{d}_o^{(hk)} - D\hat{N}^{(oo)} \tag{32}$$

lies within the observation plane. It follows that

$$(\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)}) \cdot \hat{N}_o = 0 \quad . \tag{33}$$

Using equation (32) in (33) and solving the resulting expression for $|\bar{D}_o^{(hk)}|$ yields

$$|\bar{D}_o^{(hk)}| = \frac{D}{\hat{d}_o^{(hk)} . \hat{N}_o} \tag{34}$$

With this result equation (32) becomes

$$\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)} = \frac{D\,\hat{d}_o^{(hk)}}{\hat{d}_o^{(hk)} . \hat{N}_o} - D\hat{N}_o \quad . \tag{35}$$

When equation (27) is used in equation (31) there results

$$\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)} = \frac{D\,(h\lambda\bar{a}_o{}^* + k\lambda\bar{b}_o{}^*)}{\hat{d}_o^{(hk)} . \hat{N}_o} \tag{36}$$

where $\hat{d}_o^{(hk)} . \hat{N}_o = \cos\,(\Theta_o^{(hk)}) =$

$$\sqrt{1 - h^2\lambda^2|\bar{a}_o{}^*|^2 - k^2\lambda^2|\bar{b}_o{}^*|^2 - 2(h\lambda)(k\lambda)\;\bar{a}_o{}^*.\bar{b}_o{}^*} \quad , \tag{37}$$

and $\Theta_o^{(hk)}$ is the angle between $\hat{d}_o^{(hk)}$ and the normal $\hat{N}_o$. Hence equation (36) can be rewritten as

$$\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)} = \frac{D\,(h\lambda\bar{a}_o{}^* + k\lambda\bar{b}_o{}^*)}{\cos\Theta_o^{(hk)}} \quad . \tag{38}$$

The vector $\bar{D}_o^{(hk)} - \bar{D}_o^{(hk)}$, within the observation plane, locates order (hk) with respect to the zero order. The factor $1/\cos\Theta_o^{(hk)}$ in equation (38) is a distortion factor that results from using a flat observation screen rather than a spherical one. For diffraction spots near the zero the value of this factor is nearly unity. Using equations (19) and (20) in equation (37) gives

$$\cos\Theta_o^{(hk)} = \sqrt{1 - \left(\frac{h\lambda}{P_b^o}\right)^2 - \left(\frac{k\lambda}{P_a^o}\right)^2 + 2(h\lambda)(k\lambda)\frac{\cos(\bar{a}_o,\bar{b}_o)}{P_a^o\,P_b^o}} \tag{39}$$

Equation (39) indicates that $\cos\Theta_o^{(hk)}$ is nearly unity when

$$\frac{h\lambda}{P_b^o} \ll 1 \quad \text{and} \quad \frac{k\lambda}{P_a^o} \ll 1 \quad . \tag{40}$$

For those orders (h,k) that satisfy inequalities (40) the distortion in the pattern is negligible. One can think of a "distortion" circle about the zero order within which the distortion is negligible. Outside of this circle the pattern is appreciably distorted. The radius of the circle is dictated by the degree of distortion that can be tolerated. This is illustrated in Figure 4 for an orthogonal lattice.

Within the circle of Figure 4 equation (38) reduces to

$$\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)} = D\lambda(h\bar{a}_o{}^* + k\bar{b}_o{}^*) \qquad \begin{array}{l} h = 0, \pm 1, \pm 2, \ldots, \pm m \\ k = 0, \pm 1, \pm 2, \ldots, \pm n \end{array} \tag{41}$$

According to equation (41) the locations $\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)}$ of the diffraction spots within the distortion circle are the points of the reciprocal lattice $(h\bar{a}_o{}^* + k\bar{b}_o{}^*)$, magnified by the factor $D\lambda$.

Assume that an orthogonal lattice ($\bar{a}_o$, $\bar{b}_o$) is subjected to arbitrary normal and shear strains. It is assumed that the strains are homogeneous across the lattice and that the lattice plane is not tilted by the deformation. The undeformed and deformed lattices are shown in Figure 5. A rigid translation has no effect on the diffraction pattern as seen in the focal plane of a well corrected lens. Hence, the rigid translation part of the deformation can be ignored as long as a lens is used in the formation of the pattern. The reciprocal lattices of the undeformed and deformed real lattices are shown in Figure 6. The original position of order (h,k) on the observation screen is from equation (38)

$$\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)} = \frac{D\lambda \ (h\bar{a}_o{}^* + k\bar{b}_o{}^*)}{\cos \Theta_o^{(hk)}} \tag{38}$$

The position of this diffraction spot after deformation is

$$\bar{D}^{(hk)} - \bar{D}_o^{(oo)} = \frac{D\lambda \ (h\bar{a}^* + k\bar{b}^*)}{\cos \Theta^{(hk)}} \tag{42}$$

where: $$\cos \Theta^{(hk)} = \sqrt{1 - \left(\frac{h\lambda}{P_b}\right)^2 - \left(\frac{k\lambda}{P_a}\right)^2 + \frac{2(h\lambda)(k\lambda)\ \cos(\bar{a},\ \bar{b})}{P_a\ P_b}}\ , \tag{43}$$

$\bar{a}^*$ and $\bar{b}^*$ are reciprocal vectors to the deformed lattice, and P_a and P_b are the perpendicular distances between lattice lines after deformation. The vectors $\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)}$ and $\bar{D}^{(hk)} - \bar{D}_o^{(oo)}$ are measured from the diffraction pattern. Angles $\Theta_o^{(hk)}$ and $\Theta^{(hk)}$ are also measurable quantities. They are determined from the equations

$$\tan \Theta_o^{(hk)} = \frac{\left|\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)}\right|}{D} \tag{44}$$

and $$\tan \Theta^{(hk)} = \frac{\left|\bar{D}^{(hk)} - \bar{D}_o^{(oo)}\right|}{D}\ . \tag{45}$$

If a lens is used then D is replaced by the focal length of the lens.

The extensions of the lattice vectors are

$$\varepsilon_a = \frac{|\bar{a}| - |\bar{a}_o|}{|\bar{a}_o|} = \frac{|\bar{a}|}{|\bar{a}_o|} - 1 \tag{46}$$

and
$$\varepsilon_b = \frac{|\bar{b}| - |\bar{b}_o|}{|\bar{b}_o|} = \frac{|\bar{b}|}{|\bar{b}_o|} - 1 \quad . \tag{47}$$

From equations (38) and (42)

$$\frac{\cos \Theta_o^{(ho)} \; [\bar{D}_o^{(ho)} - \bar{D}_o^{(oo)}]}{Dh\lambda} = \bar{a}_o^* \tag{48}$$

and

$$\frac{\cos \Theta^{(ho)} \; [\bar{D}^{(ho)} - \bar{D}_o^{(oo)}]}{Dh\lambda} = \bar{a}^* \tag{49}$$

For an originally orthogonal lattice $(\bar{a}_o, \bar{b}_o)$ the magnitudes of $\bar{a}_o^*$ and $\bar{a}^*$ are

$$|\bar{a}_o|^* = \frac{1}{P_b^o} = \frac{1}{|\bar{a}_o|} \tag{50}$$

$$|\bar{a}^*| = \frac{1}{P_b} = \frac{1}{|\bar{a}| \sin(\bar{a},\bar{b})} \quad . \tag{51}$$

Combining equations (48), (49), and (50) gives

$$|\bar{a}_o^*| = \frac{1}{|\bar{a}_o|} = \frac{\cos \Theta_o^{(ho)} \, |\bar{D}_o^{(ho)} - \bar{D}_o^{(oo)}|}{|Dh\lambda|} \tag{52}$$

and
$$|\bar{a}^*| = \frac{1}{|\bar{a}| \sin(\bar{a},\bar{b})} = \frac{\cos \Theta^{(ho)} \, |\bar{D}^{(ho)} - \bar{D}_o^{(oo)}|}{|Dh\lambda|} \tag{53}$$

Rewriting equation (53) gives

$$|\bar{a}| = \frac{|Dh\lambda|}{\sin(\bar{a},\bar{b}) \; \cos \Theta^{(h,o)} \, |\bar{D}^{(h,o)} - \bar{D}_o^{(oo)}|} \tag{54}$$

However, from Figure 6

$$\angle(\bar{a},\bar{b}) + \angle(\bar{a}^*,\bar{b}^*) = \pi \quad . \tag{55}$$

This equation also follows from equation (23) that was developed for a non-orthogonal lattice.

From equation (55) it follows that

$$\sin(\bar{a},\bar{b}) = \sin(\bar{a}^*,\bar{b}^*). \tag{56}$$

The extension of lattice vector $\bar{a}_o$ is obtained by combining equations (46), (52), (54) and (56) to get

$$\varepsilon_a = \frac{\cos\Theta_o^{(ho)}\,|\bar{D}_o^{(ho)} - \bar{D}_o^{(oo)}|}{\cos\Theta^{(h,o)}\,|\bar{D}^{(ho)} - \bar{D}_o^{(oo)}|\sin(\bar{a}^*,\bar{b}^*)} - 1 \ . \tag{57}$$

A similar calculation gives the extension for lattice vector $\bar{b}_o$ as

$$\varepsilon_b = \frac{\cos\Theta_o^{(ok)}\,|\bar{D}_o^{(ok)} - \bar{D}_o^{(oo)}|}{\cos\Theta^{(ok)}\,|\bar{D}^{(ok)} - \bar{D}_o^{(oo)}|\sin(\bar{a}^*,\bar{b}^*)} - 1 \ . \tag{58}$$

The shear strain is given by

$$\Gamma_{ab} = \frac{\pi}{2} - \angle(\bar{a},\bar{b}) \ . \tag{59}$$

Combining equations (55) and (59) gives

$$\Gamma_{ab} = \angle(\bar{a}^*,\bar{b}^*) - \frac{\pi}{2} \tag{60}$$

where $\angle(\bar{a}^*,\bar{b}^*)$ is measured directly from the diffraction pattern. Due to equations (44) and (45) $\cos\Theta_o^{(hk)}$ and $\cos\Theta^{(hk)}$ can be determined directly from

$$\cos\Theta_o^{(hk)} = \frac{1}{\sqrt{1 + \dfrac{|\bar{D}_o^{(hk)} - D_o^{(oo)}|^2}{D^2}}} \tag{61}$$

and $$\cos\Theta^{(hk)} = \frac{1}{\sqrt{1 + \dfrac{|\bar{D}^{(hk)} - D_o^{(oo)}|^2}{D^2}}} \tag{62}$$

Hence to compute the extension of lattice vector $\bar{a}_o$ one need only measure three quantities. These are:

$$\angle(\bar{a}^*,\bar{b}^*)\ ,$$

$$|\bar{D}_o^{(ho)} - \bar{D}_o^{(oo)}|,$$

and $$|\bar{D}^{(ho)} - \bar{D}_o^{(oo)}|.$$

To compute the extension of lattice vector $\bar{b}_o$ two additional measurements are needed. These are:

$$|\bar{D}_o^{(ok)} - \bar{D}_o^{(oo)}|\ ,$$

and $$|\bar{D}^{(ok)} - \bar{D}_o^{(oo)}|\ .$$

The necessary quantities to be measured are shown in Figure 7. Once these quantities are measured the entire state of strain can be determined.

III. THE DESCRIPTION OF RIGID ROTATIONS OF A CROSSED GRATING

The most general diffraction-strain equations should include the effects of an arbitrary finite rigid rotation of the grating. Since finite rotations cannot be treated as vector quantities it is first necessary to select coordinates which locate uniquely the angular orientation of the grating. These coordinates should provide the position of the rotated grating with respect to its original position.

Suppose that an orthogonal lattice $(\bar{a}_o,\bar{b}_o)$ suffers an arbitrary rigid rotation. Then the final position of the lattice can be specified by a set of Euler angles (ψ,θ,ϕ) as shown in Figure 8. The rotated lattice vectors are $(\bar{a},\bar{b})$ as shown. Vectors $\bar{c}_o$ and $\bar{c}$ are the normals to the lattice before and after the rotation. Angle θ is the angle of tilt of the lattice plane about line ω.

Angle ψ establishes the axis of tilt of the grating which lies in the grating plane. The total angle of rotation of the lattice about the normal is $\phi+\psi$. It is emphasized that the Euler angles determine the

final orientation of the lattice without regard to the actual sequence of rotations that produces the final configuration.

The rotation can also be described by an orthogonal matrix A that relates the unit lattice vectors of the two configurations as follows

$$\hat{u}_i = A_{ij}\hat{e}_j \qquad i,j = 1,2,3 \tag{63}$$

where $\hat{e}_i$ are original unit lattice vectors and $\hat{u}_i$ are final ones. Since matrix A and the Euler angles both describe the same rotation they must be related. To determine this relation it is convenient to take the rotation as three separate rotations. The first of these is a rotation about the normal $\bar{c}_o$ of the grating as shown in Figure 1. The unit lattice vectors of the rotated lattice are

$$\hat{e}_i' = a_{ij}\hat{e}_j \qquad i,j = 1,2,3 \tag{64}$$

where

$$a = \left[a_{ij}\right] = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} . \tag{65}$$

The second rotation is a tilt of the plane of the grating about line ω of Figure 9 as shown in Figure 10. This rotation produces unit lattice vectors $\hat{e}_i''$ as given by

$$\hat{e}_i'' = b_{ij}\hat{e}_j' \qquad i,j = 1,2,3 \tag{67}$$

where

$$b = \left[b_{ij}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} . \tag{68}$$

The final rotation is about normal $\bar{c}$ of the grating as shown in Figure 11.

Unit vectors of this configuration are

$$\hat{u}_i = c_{ij}\hat{e}_j'' \qquad i,j = 1,2,3 \tag{69}$$

where

$$c = \left[c_{ij}\right] = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} . \tag{70}$$

Combining equations (64), (67) and (69) gives

$$\hat{u}_i = c_{ij}b_{jk}a_{kt}\hat{e}_t \ . \qquad i,j,k,t = 1,2,3 \tag{71}$$

In matrix notation equation (71) becomes

$$\hat{u} = cba\hat{e} \tag{72}$$

where $\hat{u}$ and $\hat{e}$ are unit column vectors. If matrix A is defined as

$$A = cba$$

$$\text{or} \qquad A_{ij} = c_{ip}b_{pq}a_{qj} \qquad i,j,p,q = 1,2,3 \tag{73}$$

then equation (63) follows. Since the rotation matrix A is the product of the three successive rotation matrices, it follows that

$$A = \left[A_{ij}\right] = \begin{bmatrix} \cos\phi\cos\psi-\sin\phi\cos\theta\sin\psi & \cos\phi\sin\psi+\sin\phi\cos\theta\cos\psi & \sin\phi\sin\theta \\ -\sin\phi\cos\psi-\cos\phi\cos\theta\sin\psi & -\sin\phi\sin\psi+\cos\phi\cos\theta\cos\psi & \cos\phi\sin\theta \\ \sin\theta\sin\psi & -\sin\theta\cos\psi & \cos\theta \end{bmatrix} . \tag{74}$$

To produce the final configuration of the grating three successive rotations were used. However, the actual rotation of the grating into this final configuration is arbitrary. Any combination of rotations that produce the same final configuration will yield the same Euler angles and matrix A. In fact, the final orientation can be reached by two rotations in at least two different ways. The total rotation can be achieved by first a tilt of the grating plane followed by a single rotation about the grating normal. If the plane is tilted through angle θ about line ω

the unit lattice vectors become

$$\hat{e}_i' = a_{ij}'\hat{e}_j \qquad i,j = 1,2,3 \tag{75}$$

where

$$a' = \left[a_{ij}'\right] = \begin{bmatrix} \cos^2\psi+\cos\theta\sin^2\psi & \sin\psi\cos\psi(1-\cos\theta) & -\sin\theta\sin\psi \\ \sin\psi\cos\psi(1-\cos\theta) & \sin^2\psi+\cos\theta\cos^2\psi & \sin\theta\cos\psi \\ \sin\theta\sin\psi & -\sin\theta\cos\psi & \cos\theta \end{bmatrix} . \tag{76}$$

This is followed by a rotation $(\phi+\psi)$ about the new grating normal. The last rotation produces the unit lattice vectors

$$\hat{u}_i = b_{ij}'\hat{e}_j' \qquad i,j = 1,2,3 \tag{77}$$

where

$$b' = \left[b_{ij}'\right] = \begin{bmatrix} \cos(\psi+\phi) & \sin(\psi+\phi) & 0 \\ -\sin(\psi+\phi) & \cos(\psi+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} , \tag{78}$$

ψ is the angle that locates line ω, and ϕ is measured from line ω as before. The combined matrix of these two rotations is given by

$$A = b'a' \tag{79}$$

where A is given by equation (74), a' by equation (76) and b' by equation (78).

The final configuration can also be reached by first a rotation about the normal of angle $(\phi+\psi)$ followed by a rotation about line ω of angle θ. The first rotation is characterized by matrix

$$b' = \left[b_{ij}'\right] = \begin{bmatrix} \cos(\psi+\phi) & \sin(\psi+\phi) & 0 \\ -\sin(\psi+\phi) & \cos(\psi+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} . \tag{80}$$

This one is followed by a rotation about line ω with matrix

$$c' = \left[c'_{ij}\right] = \begin{bmatrix} \cos^2\psi+\cos\theta\sin^2\psi & \sin\psi\cos\psi(\cos\theta-1) & \sin\theta\sin\psi \\ \sin\psi\cos\psi(\cos\theta-1) & \sin^2\psi+\cos\theta\cos^2\psi & \sin\theta\cos\psi \\ -\sin\theta\sin\psi & -\sin\theta\cos\psi & \cos\theta \end{bmatrix}. \quad (81)$$

The total rotation matrix is then

$$A = c'b' \quad (82)$$

where A is also given by equation (74).

Regardless of the number of rotations used to achieve the final grating orientation the values of ψ, θ, and ϕ are the same. These angles are measures of the net rotation resulting from any rotation history. Angle θ is the resultant angle of tilt of the grating plane. It is the angle between the normal before rotation and the normal after rotation. Angle $(\psi+\phi)$ is the sum rotation of the lattice about the grating normal. This angle is measured about the moving normal as it is carried through the rotation process. Angle ψ establishes the axis of the resultant tilt of the grating and is designated as line ω in Figures 8, 9, 10, and 11.

The choice of Euler's angles to describe grating rotations facilitates the analysis of the diffraction problem. It will be shown that these angles can be measured directly from the diffraction pattern. In fact, angles ψ and θ can be measured by observing only the zero order of diffraction. To determine angle ϕ it is necessary to record the movement of some of the diffraction spots surrounding the zero order.

IV. THE RESPONSE OF AN ORTHOGONAL GRATING ALIGNED WITH PRINCIPAL STRAIN DIRECTIONS AND SUBJECTED TO ARBITRARY RIGID ROTATIONS

Diffraction-strain equations will be derived for an orthogonal lattice aligned with principal strain directions and subjected to arbitrary surface rotations. For moderate angles of tilt of the grating it is shown that the effect of a surface tilt is almost exactly equivalent to that of an appropriate change in the direction of the incident light. This is not the case for large angles of tilt since the plane of observation is fixed relative to the incident beam.

Consider an orthogonal lattice $(\bar{a}_o, \bar{b}_o)$ as shown in Figure 12. Vector $\bar{c}_o$ is normal to the lattice. The unit lattice vectors are $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$. The incident illumination is normal to the grating before deformation. Directions of constructive interference are determined from the equations:

$$\hat{d}_o^{(hk)} . \bar{a}_o = h\lambda \qquad h = 0, \pm 1, \pm 2, \ldots \pm m \tag{83}$$

$$\hat{d}_o^{(hk)} . \bar{b}_o = k\lambda \qquad k = 0, \pm 1, \pm 2, \ldots \pm n \tag{84}$$

where $|\hat{d}_o^{(hk)}| = 1$. (85)

The subscript o refers to the original unstrained lattice. Equations (83) and (84) can be rewritten in terms of the unit lattice vectors as

$$\hat{d}_o^{(hk)} . \hat{e}_1 = \frac{h\lambda}{|\bar{a}_o|} \qquad (83)'$$

and

$$\hat{d}_o^{(hk)} . \hat{e}_2 = \frac{k\lambda}{|\bar{b}_o|} \quad . \qquad (84)'$$

A solution for $\hat{d}_o^{(hk)}$ is found by determining $\alpha_i^{(hk)}$ that satisfy the relation

$$\hat{d}_o^{(hk)} = \alpha_i^{(hk)} \hat{e}_i = \alpha_1^{(hk)} \hat{e}_1 + \alpha_2^{(hk)} \hat{e}_2 + \alpha_3^{(hk)} \hat{e}_3 \quad . \tag{86}$$

Using equation (86) in equation (83)' and (84)' gives

$$\alpha_1^{(hk)} = \frac{h\lambda}{|\bar{a}_o|} \tag{87}$$

and $$\alpha_2^{(hk)} = \frac{k\lambda}{|\bar{b}_o|} \quad . \tag{88}$$

Combining equations (85) and (86) gives

$$|\hat{d}_o^{(hk)}|^2 = 1 = [\alpha_1^{(hk)}]^2 + [\alpha_2^{(hk)}]^2 + [\alpha_3^{(hk)}]^2 \tag{89}$$

Solving equation (89) for $\alpha_3^{(hk)}$ yields

$$\alpha_3^{(hk)} = + \sqrt{1 - [\alpha_1^{(hk)}]^2 - [\alpha_2^{(hk)}]^2} \tag{90}$$

where the plus sign is chosen for a reflection grating. Using equations (87), (88) and (90) in equation (86) there results

$$\hat{d}_o^{(hk)} = \frac{h\lambda}{|\bar{a}_o|}\hat{e}_1 + \frac{k\lambda}{|\bar{b}_o|}\hat{e}_2 + \sqrt{1 - \left(\frac{h\lambda}{|\bar{a}_o|}\right)^2 - \left(\frac{k\lambda}{|\bar{b}_o|}\right)^2}\,\hat{e}_3 \quad . \tag{91}$$

Equation (91) determines the various directions of constructive interference before deformation.

It is assumed that the lines of the crossed grating are ruled in the anticipated directions of the principal surface strains. When this is the case the deformation will preserve the orthogonality of the grating. In addition to the normal extensions the lattice suffers arbitrary rigid rotations. The directions of constructive interference at any time during the deformation are determined by the equations

$$(\hat{d}^{(hk)} - \hat{i}_o) \cdot \hat{u}_1 = \frac{h\lambda}{|\bar{a}|} \qquad h = 0, \pm 1, \ldots \pm m, \tag{92}$$

$$(\hat{d}^{(hk)} - \hat{i}_o) \cdot \hat{u}_2 = \frac{k\lambda}{|\bar{b}|} \qquad k = 0, \pm 1, \ldots \pm n, \tag{93}$$

$$|\hat{d}^{(hk)}| = 1 \ , \tag{94}$$

$$|\hat{i}_o| = 1 \ , \tag{95}$$

where $\hat{i}_o$ is a unit vector in the fixed direction of the incident light, $\hat{u}_1$ and $\hat{u}_2$ are the current unit lattice vectors, and $|\bar{a}|$ and $|\bar{b}|$ are the magnitudes of the respective lattice vectors. Figure 13 shows the deformed lattice referred to the original unit lattice $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$. Vector $\hat{d}^{(hk)}$ defines the direction of order (hk) as the grating is being deformed. To solve equations (92) through (95) it is assumed that

$$\hat{d}^{(hk)} = \beta_i^{(hk)}\hat{u}_i = \beta_1^{(hk)}\hat{u}_1 + \beta_2^{(hk)}\hat{u}_2 + \beta_3^{(hk)}\hat{u}_3 \tag{96}$$

where $\beta_i^{(hk)}$ are to be determined. Using equation (96) in equations (92) through (94) there results

$$\beta_1^{(hk)} = \hat{i}_o\cdot\hat{u}_1 + \frac{h\lambda}{|\bar{a}|}, \tag{97}$$

$$\beta_2^{(hk)} = \hat{i}_o\cdot\hat{u}_2 + \frac{k\lambda}{|\bar{b}|}, \tag{98}$$

and $$|\hat{d}_o^{(hk)}|^2 = 1 = [\beta_1^{(hk)}]^2 + [\beta_2^{(hk)}]^2 + [\beta_3^{(hk)}]^2 \tag{99}$$

Solving equation (99) for $\beta_3^{(hk)}$ yields

$$\beta_3^{(hk)} = +\sqrt{1 - [\beta_1^{(hk)}]^2 - [\beta_2^{(hk)}]^2}\ . \tag{100}$$

Combining equations (96) through (100) there results

$$\hat{d}^{(hk)} = (\hat{i}_o\cdot\hat{u}_1 + \frac{h\lambda}{|\bar{a}|})\hat{u}_1 + (\hat{i}_o\cdot\hat{u}_2 + \frac{k\lambda}{|\bar{b}|})\hat{u}_2 + \sqrt{1 - (\hat{i}_o\cdot\hat{u}_1 + \frac{h\lambda}{|\bar{a}|})^2 - (\hat{i}_o\cdot\hat{u}_2 + \frac{k\lambda}{|\bar{b}|})^2}\,\hat{u}_3\ . \tag{101}$$

Equation (101) provides the directions of constructive interference during deformation in terms of the rotated unit lattice vectors.

The plane of observation is placed parallel to the original grating plane at a distance D away. When a lens is used in the formation of the diffraction pattern, the observation plane corresponds to the focal plane of the lens. In this case D is replaced by the focal length. Vector $\bar{D}_o^{(hk)}$

to the observation plane locates the diffraction spot of order (hk).

This vector is related to $\hat{d}_o^{(hk)}$ by

$$\bar{D}_o^{(hk)} = \frac{D\hat{d}_o^{(hk)}}{\hat{d}_o^{(hk)} \cdot \hat{e}_3} = \frac{D\hat{d}_o^{(hk)}}{\cos\Theta_o^{(hk)}} , \tag{102}$$

where $\Theta_o^{(hk)}$ is the angle between $\hat{d}_o^{(hk)}$ and $\hat{e}_3$. The observation screen and vectors $\bar{D}_o^{(oo)}$ and $\bar{D}_o^{(hk)}$ are shown in Figure 3. From equation (38) it follows that

$$\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)} = \frac{D\left(\frac{h\lambda}{|\bar{a}_o|}\hat{e}_1 + \frac{k\lambda}{|\bar{b}_o|}\hat{e}_2\right)}{\cos\Theta_o^{(hk)}} \tag{103}$$

where

$$\cos\Theta_o^{(hk)} = \sqrt{1 - \frac{h^2\lambda^2}{|\bar{a}_o|^2} - \frac{k^2\lambda^2}{|\bar{b}_o|^2}} . \tag{104}$$

This vector lies in the observation plane and joins orders (hk) and (oo). As a result of deformation the diffraction spots will move to new locations defined by vector $\bar{D}^{(hk)}$. This vector is related to $\hat{d}^{(hk)}$ by

$$\bar{D}^{(hk)} = \frac{D\hat{d}^{(hk)}}{\hat{d}^{(hk)} \cdot \hat{e}_3} \tag{105}$$

where $\hat{d}^{(hk)}$ is given in equation (101). It is necessary to find $\hat{d}^{(hk)}$ in terms of the original unit lattice vectors $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$. The unit lattice vectors before and during deformation are related by

$$\hat{u}_i = A_{ij}\hat{e}_j \qquad i,j = 1,2,3 \tag{106}$$

where A is the rotation matrix given in equation (74). When equation (106) is used in equation (96) there results

$$\hat{d}^{(hk)} = \beta_i^{(hk)}\hat{u}_i = \beta_i^{(hk)}A'_{ij}\hat{e}_j . \qquad i,j = 1,2,3 \tag{107}$$

This can be rewritten as

$$\hat{d}^{(hk)} = \beta_j'^{(hk)} \hat{e}_j \tag{108}$$

where

$$\beta_j'^{(hk)} = \beta_i^{(hk)} A_{ij} . \tag{109}$$

If equation (108) is used in equation (105) the result is

$$\bar{D}^{(hk)} = \frac{D\beta_j'^{(hk)} \hat{e}_j}{\beta_3'^{(hk)}} . \tag{110}$$

Now vector $\bar{D}^{(hk)} - \bar{D}_o^{(oo)}$ will lie within the observation plane as shown in Figure 14. Due to equation (110) this vector is

$$\bar{D}^{(hk)} - \bar{D}_o^{(oo)} = \frac{D\beta_j'^{(hk)} \hat{e}_j}{\beta_3'^{(hk)}} - D\hat{e}_3 = \frac{D(\beta_1'^{(hk)} \hat{e}_1 + \beta_2'^{(hk)} \hat{e}_2)}{\beta_3'^{(hk)}} . \tag{111}$$

Equation (111) can be written as

$$\frac{\cos\Theta^{(hk)}(\bar{D}^{(hk)} - \bar{D}_o^{(oo)})}{D} = (\beta_1'^{(hk)} \hat{e}_1 + \beta_2'^{(hk)} \hat{e}_2) \tag{112}$$

where $\beta_3'^{(hk)} = \cos\Theta^{(hk)}$ and $\Theta^{(hk)}$ is the angle between order (hk) during deformation and the original zero order before deformation. From equation (112) the new position of the zero order is

$$\frac{\cos\Theta^{(oo)}(\bar{D}^{(oo)} - \bar{D}_o^{(oo)})}{D} = (\beta_1'^{(oo)} \hat{e}_1 + \beta_2'^{(oo)} \hat{e}_2) . \tag{113}$$

Equation (103) is rewritten as

$$\cos\Theta_o^{(hk)} \left[\frac{\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)}}{D} \right] = \left(\frac{h\lambda}{|\bar{a}_o|} \hat{e}_1 + \frac{k\lambda}{|\bar{b}_o|} \hat{e}_2 \right) . \tag{114}$$

The factor $\cos\Theta_o^{(hk)}$ in equation (114) removes the distortion in vector $(\bar{D}_o^{(hk)} - \bar{D}_o^{(oo)})$ which is caused by a plane observation screen. Subtracting equation (113) from (112) gives

$$\frac{\cos\Theta^{(hk)}\ (\bar{D}^{(hk)} - \bar{D}_o^{(oo)})}{D} - \frac{\cos\Theta^{(oo)}\ (\bar{D}^{(oo)} - \bar{D}_o^{(oo)})}{D} =$$

$$[\beta_1'^{(hk)} - \beta_1'^{(oo)}]\hat{e}_1 + [\beta_2'^{(hk)} - \beta_2'^{(oo)}]\hat{e}_2 \qquad (115)$$

The cosine factors in equation (115) can also be thought of as distortion factors. From equation (109)

$$\beta_1'^{(hk)} = \beta_i^{(hk)} A_{i1} \qquad i = 1,2,3 \qquad (116)$$

and

$$\beta_2'^{(hk)} = \beta_i^{(hk)} A_{i2} \qquad i = 1,2,3 \qquad (117)$$

If equations (116) and (117) are used in equation (115) the result is

$$\frac{\cos\Theta^{(hk)}\ (\bar{D}^{(hk)} - \bar{D}_o^{(oo)})}{D} - \frac{\cos\Theta^{(oo)}\ (\bar{D}^{(oo)} - \bar{D}_o^{(oo)})}{D} =$$

$$(\beta_i^{(hk)} - \beta_i^{(oo)})A_{i1}\hat{e}_1 + (\beta_i^{(hk)} - \beta_i^{(oo)})A_{i2}\hat{e}_2 \ . \qquad i = 1,2,3 \qquad (118)$$

Due to equations (97), (98) and (100) it follows that

$$\beta_1^{(hk)} - \beta_1^{(oo)} = \frac{h\lambda}{|\bar{a}|} \qquad (119)$$

$$\beta_2^{(hk)} - \beta_2^{(oo)} = \frac{k\lambda}{|\bar{b}|} \qquad (120)$$

and

$$\beta_3^{(hk)} - \beta_3^{(oo)} = \sqrt{1 - (\hat{i}_o\cdot\hat{u}_1 + \frac{h\lambda}{|\bar{a}|})^2 - (\hat{i}_o\cdot\hat{u}_2 + \frac{k\lambda}{|\bar{b}|})^2} - \sqrt{1 - (\hat{i}_o\cdot\hat{u}_1)^2 - (\hat{i}_o\cdot\hat{u}_2)^2} \ . \qquad (121)$$

Expanding equation (118) and making use of equations (119) through (121) in the result gives

$$\frac{\cos\Theta^{(hk)}(\bar{D}^{(hk)} - \bar{D}_o^{(oo)})}{D} - \frac{\cos\Theta^{(oo)}(\bar{D}^{(oo)} - \bar{D}_o^{(oo)})}{D} =$$

$$\Big[\frac{h\lambda}{|\bar{a}|}(\cos\phi\cos\psi - \sin\phi\cos\theta\sin\psi) + \frac{k\lambda}{|\bar{b}|}(-\sin\phi\cos\psi - \cos\phi\cos\theta\sin\psi)$$

$$+ \left(\sqrt{1 - (\hat{i}_o\cdot\hat{u}_1 + \frac{h\lambda}{|\bar{a}|})^2 - (\hat{i}_o\cdot\hat{u}_2 + \frac{k\lambda}{|\bar{b}|})^2} - \sqrt{1 - (\hat{i}_o\cdot\hat{u}_1)^2 - (\hat{i}_o\cdot\hat{u}_2)^2}\right) X$$

$$\left.\sin\psi\sin\theta\right]\hat{e}_1 + \left[\frac{h\lambda}{|\bar{a}|}(\cos\phi\sin\psi+\sin\phi\cos\theta\cos\psi) + \frac{k\lambda}{|\bar{b}|}(-\sin\phi\sin\psi+\cos\phi\cos\theta\cos\psi)\right.$$

$$+\left(\sqrt{1-(\hat{i}_o\cdot\hat{u}_1+\frac{h\lambda}{|\bar{a}|})^2-(\hat{i}_o\cdot\hat{u}_2+\frac{k\lambda}{|\bar{b}|})^2}-\sqrt{1-(\hat{i}_o\cdot\hat{u}_1)^2-(\hat{i}_o\cdot\hat{u}_2)^2}\right)X$$

$$\left.(-\sin\theta\cos\psi)\right]\hat{e}_2\,. \tag{122}$$

Since the light is originally normal to the grating

$$\hat{i}_o = -\hat{e}_3\,. \tag{123}$$

It follows that

$$\hat{i}_o\cdot\hat{u}_i = \hat{e}_3\cdot\hat{u}_i = \hat{e}_3\cdot A_{ij}\hat{e}_j = -A_{i3}. \qquad i = 1,2,3 \tag{124}$$

From equation (124) and equation (74)

$$\hat{i}_o\cdot\hat{u}_1 = -A_{13} = -\sin\phi\sin\theta \tag{125}$$

and $$\hat{i}_o\cdot\hat{u}_2 = -A_{23} = -\cos\phi\sin\theta \tag{126}$$

Using equations (125) and (126) in equation (121) gives

$$\beta_3^{(hk)} - \beta_3^{(oo)} = \sqrt{1-(\frac{h\lambda}{|\bar{a}|}-\sin\phi\sin\theta)^2-(\frac{k\lambda}{|\bar{b}|}-\cos\phi\sin\theta)^2} - \sqrt{1-(\sin\phi\sin\theta)^2-(\cos\phi\sin\theta)^2} \tag{127}$$

For sufficiently small values of $\sin\theta$ the value of $\beta_3^{(hk)} - \beta_3^{(oo)}$ will be small provided that the following inequalities are satisfied for order (hk):

$$\left.\begin{aligned}\frac{h\lambda}{|\bar{a}|} &\ll 1\\ \frac{k\lambda}{|\bar{b}|} &\ll 1\end{aligned}\right\} \tag{128}$$

These inequalities are satisfied provided that the line spacing is large compared to λ and if those orders near the zero are used.

Since $$\sqrt{1-\chi} \simeq 1-\frac{\chi}{2} \tag{129}$$

for χ small compared to 1, equation (127) is approximated by

$$\beta_3^{(hk)} - \beta_3^{(oo)} \approx \sin\phi\sin\theta\frac{h\lambda}{|\bar{a}|} + \cos\phi\sin\theta\frac{k\lambda}{|\bar{b}|} - \frac{h^2\lambda^2}{2|\bar{a}|^2} - \frac{k^2\lambda^2}{2|\bar{b}|^2}\,. \tag{130}$$

Using equation (130) in equation (122) yields

$$\frac{\cos\Theta^{(hk)}\,(\bar{D}^{(hk)} - \bar{D}_o^{(oo)})}{D} - \frac{\cos\Theta^{(oo)}\,(\bar{D}^{(oo)} - \bar{D}_o^{(oo)})}{D} =$$

$$\Bigg[\frac{h\lambda}{|\bar{a}|}\left(\cos\phi\cos\psi - \sin\phi\cos\theta\sin\psi\right) + \frac{k\lambda}{|\bar{b}|}\left(-\sin\phi\cos\psi - \cos\phi\cos\theta\sin\psi\right)$$
$$+ \left(\sin\phi\sin\theta\frac{h\lambda}{|\bar{a}|} + \cos\phi\sin\theta\frac{k\lambda}{|\bar{b}|} - \frac{h^2\lambda^2}{2|\bar{a}|^2} - \frac{k^2\lambda^2}{2|\bar{b}|^2}\right)\sin\psi\sin\theta\Bigg]\hat{e}_1$$
$$+ \Bigg[\frac{h\lambda}{|\bar{a}|}\left(\cos\phi\sin\psi + \sin\phi\cos\theta\cos\psi\right) + \frac{k\lambda}{|\bar{b}|}\left(-\sin\phi\sin\psi + \cos\phi\cos\theta\cos\psi\right)$$
$$+ \left(\sin\phi\sin\theta\frac{h\lambda}{|\bar{a}|} + \cos\phi\sin\theta\frac{k\lambda}{|\bar{b}|} - \frac{h^2\lambda^2}{2|\bar{a}|^2} - \frac{k^2\lambda^2}{2|\bar{b}|^2}\right)(-\sin\theta\cos\psi)\Bigg]\hat{e}_2 \tag{131}$$

Equation (131) simplifies when the angle of tilt θ is small enough so that

$$\left.\begin{aligned} \sin\theta &\approx \theta \\ \text{and}\quad \cos\theta &\approx 1. \end{aligned}\right\} \tag{132}$$

Equation (132) are satisfied with less than one percent error for values of Θ less than eight degrees (0.14 radians).

For this case equation (131) reduces to

$$\frac{\cos\Theta^{(hk)}\,(\bar{D}^{(hk)} - \bar{D}_o^{(oo)})}{D} - \frac{\cos\Theta^{(oo)}\,(\bar{D}^{(oo)} - \bar{D}_o^{(oo)})}{D} =$$

$$\left[\frac{h\lambda}{|\bar{a}|}\cos(\phi+\psi) - \frac{k\lambda}{|\bar{b}|}\sin(\phi+\psi)\right]\hat{e}_1 + \left[\frac{h\lambda}{|\bar{a}|}\sin(\phi+\psi) + \frac{k\lambda}{|\bar{b}|}\cos(\phi+\psi)\right]\hat{e}_2 \tag{133}$$

where terms that contain θ^2, $\frac{h^2\lambda^2}{|\bar{a}|^2}$, and $\frac{k^2\lambda^2}{|\bar{b}|^2}$ are omitted.

Equation (133) can be used to obtain diffraction strain relations for angles of tilt that satisfy equations (132) and orders (hk) that satisfy inequalities (128). These assumptions would lead to negligible error in computing the position of the diffraction spot of the second order from a grating of 5000 lines per inch with a surface tilt of eight degrees.

In using equation (133) one is not restricted to small angles of rotation about the grating normal. However, the angle of tilt must satisfy equation (132).

The extensions of lattice vectors $\bar{a}_o$ and $\bar{b}_o$ are

$$\varepsilon_a = \frac{|\bar{a}| - |\bar{a}_o|}{|\bar{a}_o|} = \frac{|\bar{a}|}{|\bar{a}_o|} - 1 \tag{134}$$

and

$$\varepsilon_b = \frac{|\bar{b}| - |\bar{b}_o|}{|\bar{b}_o|} = \frac{|\bar{b}|}{|\bar{b}_o|} - 1. \tag{135}$$

The values of $|\bar{a}_o|$ and $|\bar{b}_o|$ are determined from the original diffraction pattern. From equation (114)

$$\frac{1}{|\bar{a}_o|} = \frac{\cos\Theta_o^{(ho)}\,|\bar{D}_o^{(ho)} - \bar{D}_o^{(oo)}|}{Dh\lambda} \tag{136}$$

and

$$\frac{1}{|\bar{b}_o|} = \frac{\cos\Theta_o^{(ok)}\,|\bar{D}_o^{(ok)} - \bar{D}_o^{(oo)}|}{Dk\lambda}, \tag{137}$$

where

$$\cos\Theta_o^{(ho)} = \frac{1}{\sqrt{1 + \dfrac{|\bar{D}_o^{(ho)} - \bar{D}_o^{(oo)}|^2}{D^2}}} \tag{138}$$

and

$$\cos\Theta_o^{(ok)} = \frac{1}{\sqrt{1 + \dfrac{|\bar{D}_o^{(ok)} - \bar{D}_o^{(oo)}|^2}{D^2}}} \tag{139}$$

The value of $|\bar{a}|$ is determined from equation (133) by considering order (ho). From equation (133)

$$\frac{\cos\Theta^{(ho)}\,(\bar{D}^{(ho)} - \bar{D}_o^{(oo)})}{D} - \frac{\cos\Theta^{(oo)}\,(\bar{D}^{(oo)} - \bar{D}_o^{(oo)})}{D} =$$

$$\frac{h\lambda}{|\bar{a}|}\ \text{[illegible]}\ \hat{e}_1 + \frac{h\lambda}{|\bar{a}|}\sin(\phi+\psi)\,\hat{e}_2 \ . \tag{140}$$

The individual terms on the left hand side of equation (140) are determined from measurements of the spacing of the diffraction pattern. Taking the magnitude of both sides of equation (140) and solving the resulting expression for $|\bar{a}|$ gives

$$|\bar{a}| = \frac{Dh\lambda}{\left| \cos\Theta^{(ho)} \left(\bar{D}^{(ho)} - \bar{D}_o^{(oo)}\right) - \cos\Theta^{(oo)} \left(\bar{D}^{(oo)} - \bar{D}_o^{(oo)}\right) \right|} . \quad (141)$$

A similar calculation gives

$$|\bar{b}| = \frac{Dk\lambda}{\left| \cos\Theta^{(ok)} \left(\bar{D}^{(ok)} - \bar{D}_o^{(oo)}\right) - \cos\Theta^{(oo)} \left(\bar{D}^{(oo)} - \bar{D}_o^{(oo)}\right) \right|} . \quad (142)$$

Using equations (136), (137), (141), and (142) in equations (134) and (135) yields the lattice extensions

$$\varepsilon_a = \frac{\cos\Theta_o^{(ho)} \left|\bar{D}_o^{(ho)} - \bar{D}_o^{(oo)}\right|}{\left| \cos\Theta^{(ho)} \left(\bar{D}^{(ho)} - \bar{D}_o^{(oo)}\right) - \cos\Theta^{(oo)} \left(\bar{D}^{(oo)} - \bar{D}_o^{(oo)}\right) \right|} - 1 \quad (143)$$

and

$$\varepsilon_b = \frac{\cos\Theta_o^{(ok)} \left|\bar{D}_o^{(ok)} - \bar{D}_o^{(oo)}\right|}{\left| \cos\Theta^{(ok)} \left(\bar{D}^{(ok)} - \bar{D}_o^{(oo)}\right) - \cos\Theta^{(oo)} \left(\bar{D}^{(oo)} - \bar{D}_o^{(oo)}\right) \right|} - 1 , \quad (144)$$

The distortion factors appearing in the denominators of equations (143) and (144) are determined from measurements of the spacing of diffraction spots together with the equation

$$\cos\Theta^{(hk)} = \frac{1}{\sqrt{1 + \dfrac{\left|\bar{D}^{(hk)} - \bar{D}_o^{(oo)}\right|^2}{D^2}}} . \quad (145)$$

Equation (122) would predict the position of the diffraction spots for large angles of tilt of the grating plane. For angles of tilt that

satisfy equations (132) very little error results in using the simplified equation (133).

The Euler angles of the surface rotation are determined by measurements of the spacing of the diffraction spots together with the equations

$$\tan\psi = \frac{(\bar{D}^{(oo)} - \bar{D}_o^{(oo)})_1}{-(\bar{D}^{(oo)} - \bar{D}_o^{(oo)})_2} , \tag{146}$$

$$\tan 2\theta = \frac{|\bar{D}^{(oo)} - \bar{D}_o^{(oo)}|}{D} , \tag{147}$$

and

$$\tan(\psi+\phi) = \frac{\left[\cos\Theta^{(ho)} (\bar{D}^{(ho)} - \bar{D}_o^{(oo)}) - \cos\Theta^{(oo)} (\bar{D}^{(oo)} - \bar{D}_o^{(oo)})\right]_2}{\left[\cos\Theta^{(ho)} (\bar{D}^{(ho)} - \bar{D}_o^{(oo)}) - \cos\Theta^{(oo)} (\bar{D}^{(oo)} - \bar{D}_o^{(oo)})\right]_1} \tag{148}$$

where the subscripts 1 and 2 denote the respective components of the vector bearing the subscript. Equations (146) and (148) together with Figure 15 indicate that angles ψ and θ can be measured by observing the movement of only the zero diffraction order. To determine angle ϕ it is necessary to observe at least one other diffraction spot.

It should be noted that one could construct a net having the general appearance of Figure 4 that could be used in conjunction with a recorded diffraction pattern to measure the rotation angles of the grating. This finely divided net would be drawn for a fixed grating to screen distance or equivalently a fixed focal length. The physical dimensions of the net coordinates would be degrees of arc or radians. Such a net is shown in Figure 16 and corresponds to the Greninger chart for x-ray diffraction.

In using the net one would simply lay a transparency of the recorded diffraction pattern over the net. By aligning a row of zero diffraction orders (h,o) or (o,k) with the horizontal curves of the net one could

determine the orientation of each lattice vector $\bar{a}$ and $\bar{b}$, following the method used to find the orientation of a zone axis in x-ray crystallography (1). The angles involved in this method are not identical with the Euler angles used in the foregoing discussion.

It can be shown that equation (133), which was obtained from equation (131) by neglecting higher order terms, is the exact solution to a different diffraction problem. This equivalent problem corresponds to having a zero tilt of the grating plane but a change in the direction of incidence so as to produce the same zero diffraction spot on the observation screen as would result from a tilt of the grating. The equivalent problems are shown in Figure 17. In this figure the plane of incidence is shown as the plane of the paper. The positions of the various diffraction spots on the observation screen for the equivalent problems are very nearly the same as long as the angles of tilt are small. For large angles of tilt the two problems are not equivalent. The diffraction spots would not occupy the same positions when the tilt is large. In this case equation (133) cannot be used since the higher order terms are no longer negligible. It should be noted that the reason the two problems are not equivalent for large angles of tilt is the use of a fixed observation screen or camera position. If the observation screen were rotated in such a fashion as to remain parallel to the grating as it is tilted then the two problems would be identical. However, this would be impossible to do during a dynamic experiment of short duration.

V. STRAIN RESOLUTION OF A DIFFRACTION GRATING

The Rayleigh criterion will be used to find a theoretical estimate of the strain resolving power of an optical diffraction grating. According to the Rayleigh criterion two diffraction spots are barely resolved when the intensity maximum of one falls on the first zero intensity point of the other. The situation is illustrated in Figure 18. In the figure $\Delta\theta$ is the half angular width of the diffraction order. This angular width is given by

$$\Delta\theta_n = \frac{\lambda m}{N\cos\theta_n} \tag{149}$$

where λ = wave length

m = reciprocal line spacing

N = total number of lines in the grating

θ_n = diffraction angle of the n^{th} order.

For a derivation of equation (149) see Longhurst (9). This equation is not restricted to normal incidence. The shift in the n^{th} diffraction order caused by a strain ϵ is

$$\Delta\theta_n = \frac{-n\lambda m\varepsilon}{\cos\theta_n} \tag{150}$$

where n is the order number. For a discussion of equation (150) see Douglas _et al_. (6). The smallest shift resolvable is the value given by equation (149). Hence it follows that

$$-\frac{\lambda m}{N\cos\theta_n} = \frac{n\lambda m\varepsilon}{\cos\theta_n} . \tag{151}$$

By solving equation (151) for $|\varepsilon|$ one obtains an expression for the smallest strain that can be resolved. This result is

$$|\varepsilon|_{min} = \frac{1}{Nn} . \tag{152}$$

The strain resolving power is then defined as

$$R_\varepsilon = Nn. \tag{153}$$

Equations (152) and (153) predict the inherent ability of the nth diffraction order to resolve a strain. The chromatic resolving power for wave length λ and order n is

$$\lambda/\Delta\lambda = R_\lambda = Nn. \tag{154}$$

Thus the grating can resolve a strain or change in wave length with equal ability. The total number of lines in the diffraction grating strain gage is

$$N = L_g m \tag{155}$$

where L_g is the length of the gage. Using this result equations (152) and (153) become

$$|\varepsilon|_{min} = \frac{1}{nmL_g} \tag{156}$$

and

$$R_\varepsilon = nmL_g. \tag{157}$$

For example, a grating with reciprocal spacing 10,000 lines/in. and length of 0.1 in. can resolve a strain of 0.1% in the first order. The corresponding resolving power of this grating would be 1000 for the first order of diffraction.

It is emphasized that the resolution as obtained by the Rayleigh criterion is a theorteical estimate only. Due to variations in line spacing real gratings will not actually resolve the strains predicted by equation (156). Grating irregularities caused by large strains also contribute to a decrease in resolving power. Hence, resolution values obtained by the Rayleigh criterion should be taken as limiting values only.

VI. CONCLUSIONS

Diffraction-strain equations were first developed for a crossed grating subjected to arbitrary surface strains in the absence of surface tilt. From the derived equations it is seen that one need only observe the displacements of two active orders to determine the entire state of surface strain, since the zero order would not be displaced in the absence of surface tilt. A direct measurement of the shear angle can be made from a record of the diffraction pattern.

Rigid rotations of the grating plane were considered. Euler angles were introduced as coordinates of the angular orientation of the grating. These angles were shown to be independent of the actual sequence of rotations involved, but were directly related to the tilt of the grating and the rotation about the grating normal. Equations were provided for determining the Euler angles from measurements of the spacing of the diffraction pattern.

Diffraction-strain equations were derived for an orthogonal grating aligned with principal strain directions including the effects of rigid rotations. The effect of a surface tilt was found to be the same as a change in the direction of incidence for small angles of tilt. For large angles of tilt and a fixed observation screen this would not be the case.

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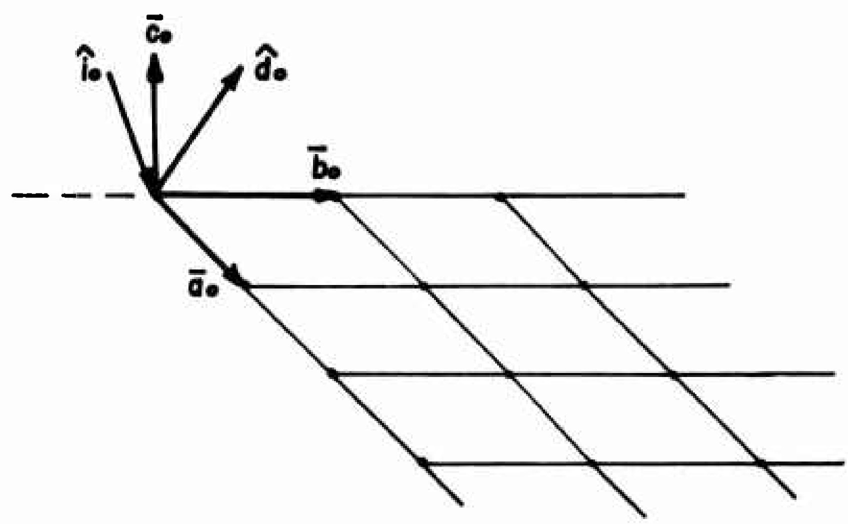

Fig. 1 Undeformed lattice

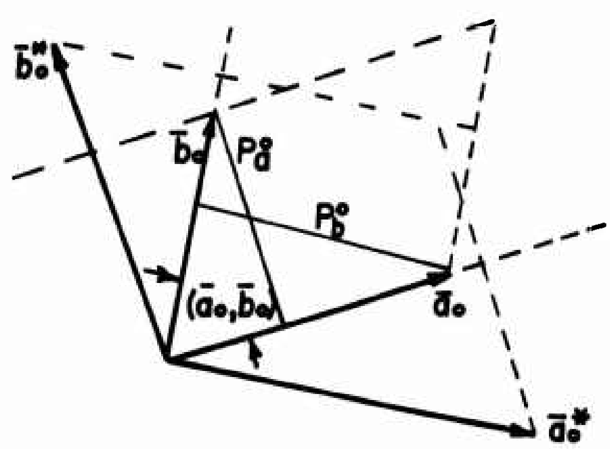

Fig. 2 Lattice and reciprocal lattice

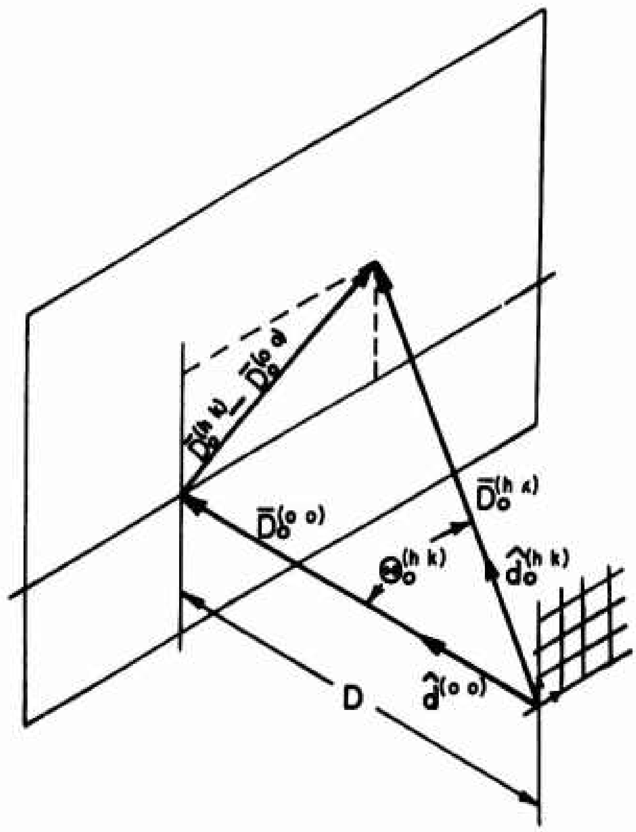

Fig. 3 Observation screen

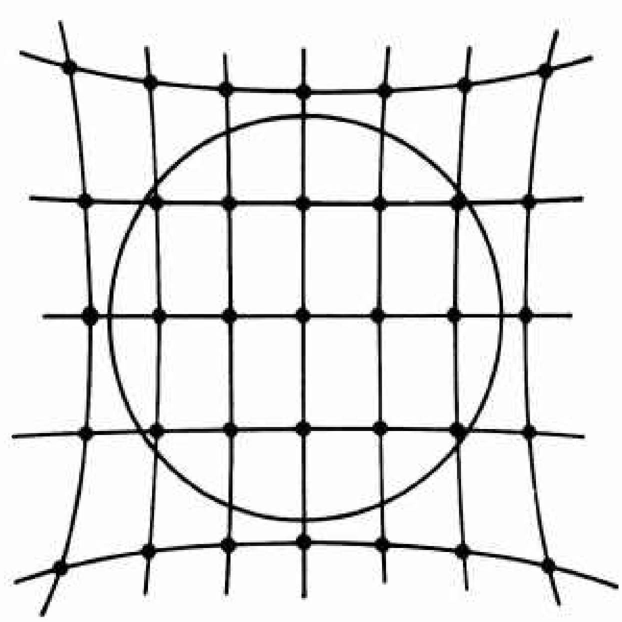

Fig. 4 Distortion circle

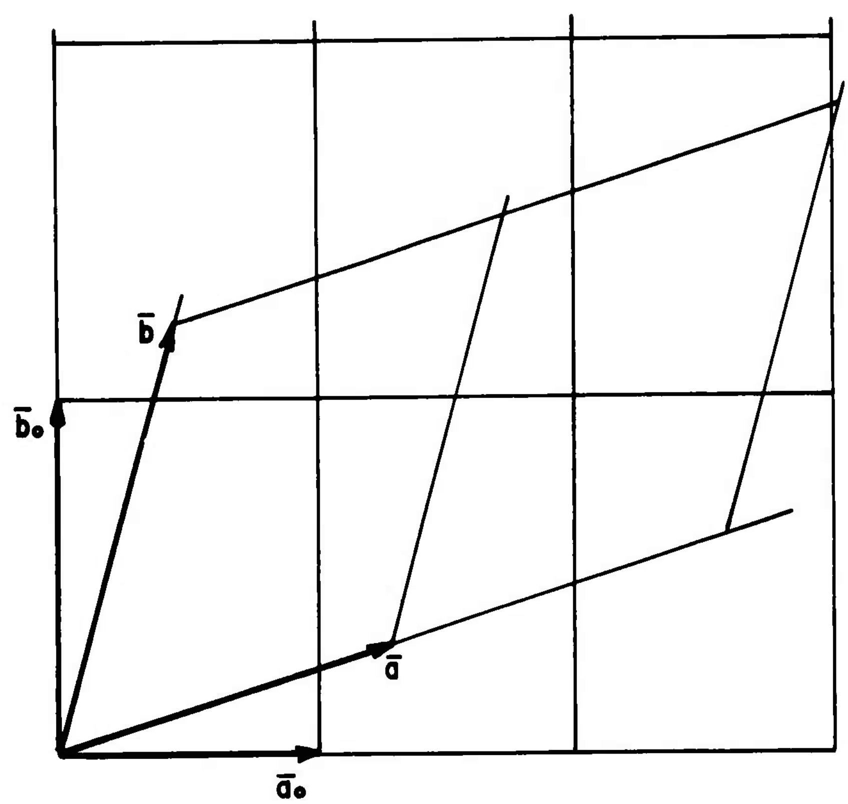

Fig. 5 Deformed and undeformed lattices

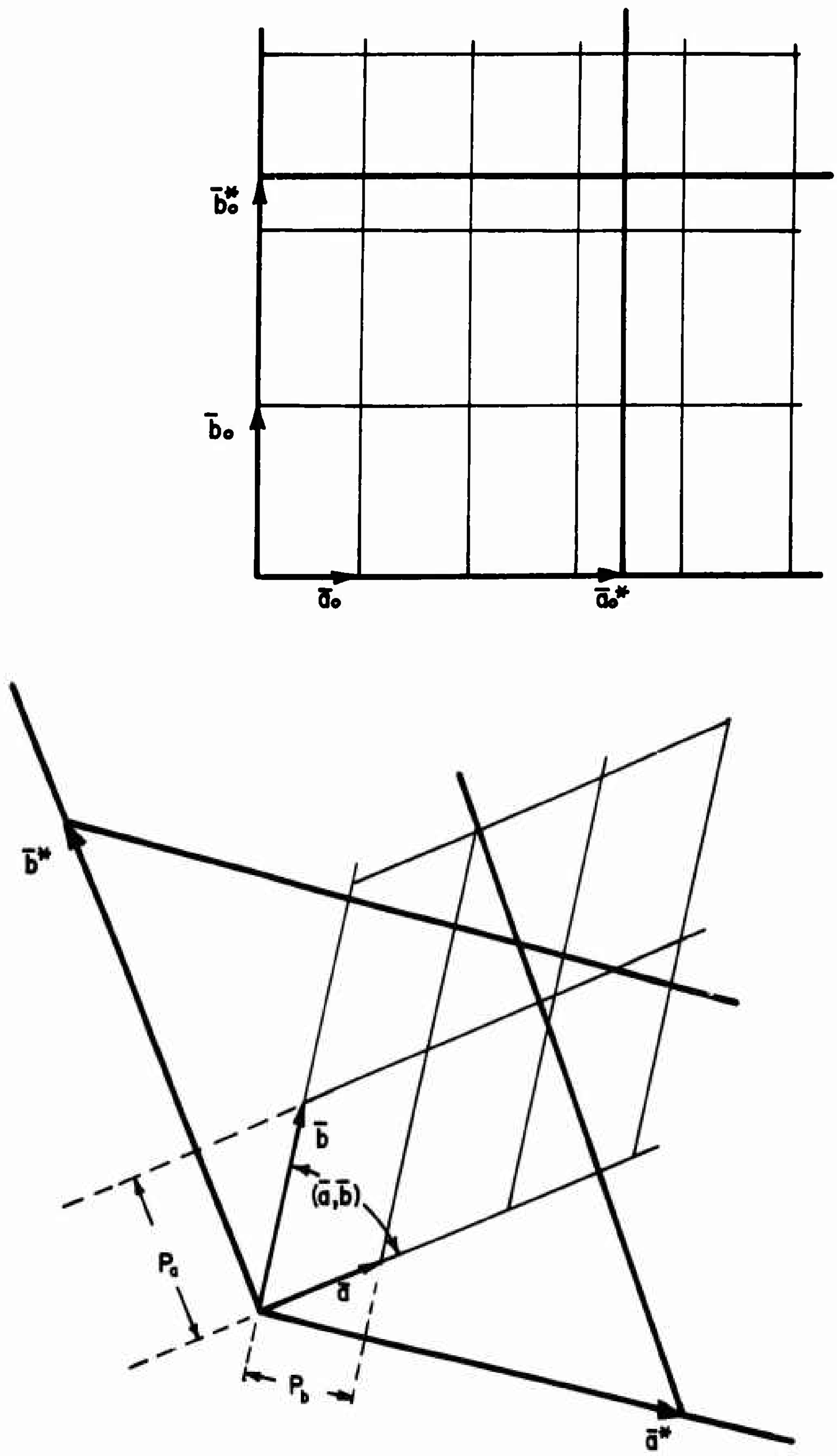

Fig. 6 Reciprocal lattices before and after deformation

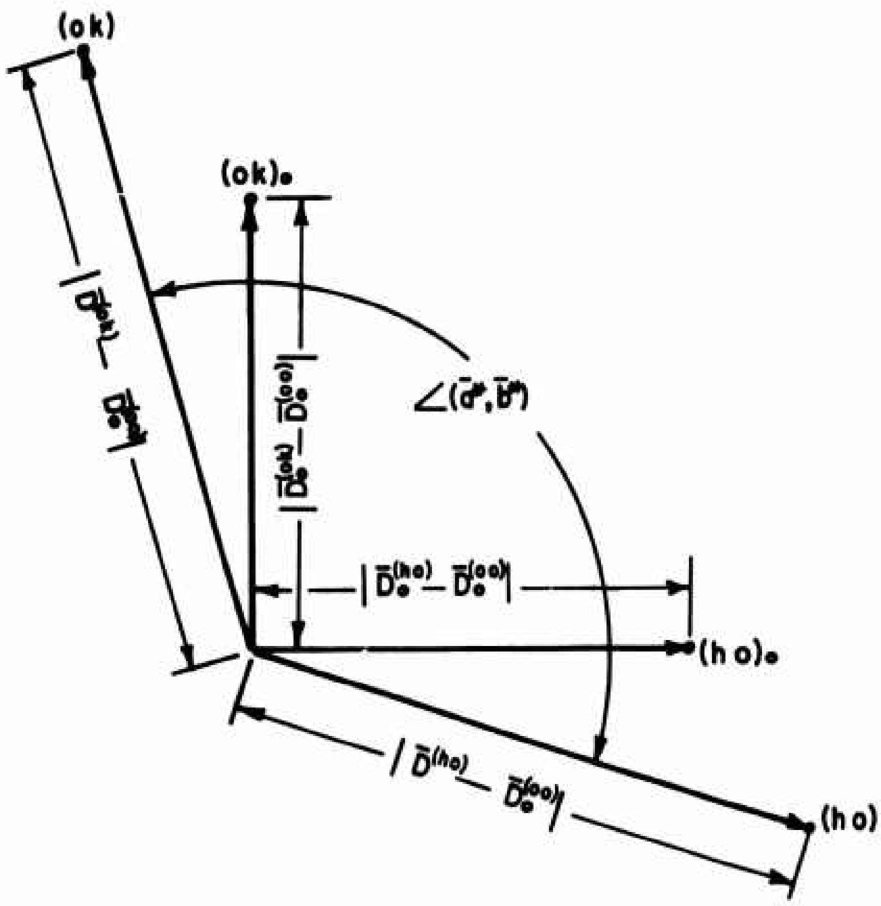

Fig. 7 Quantities to be measured

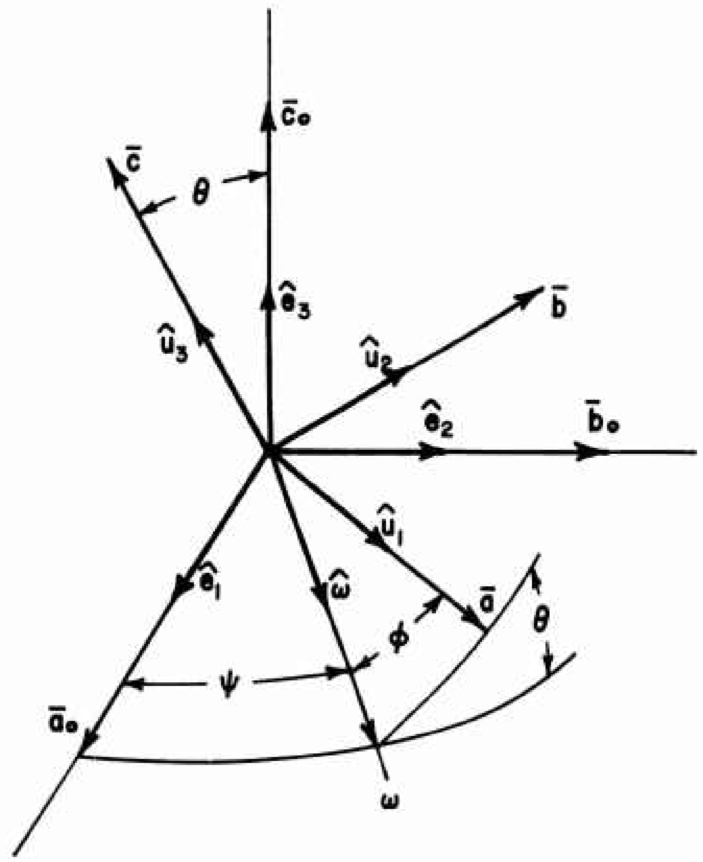

Fig. 8 Euler angles

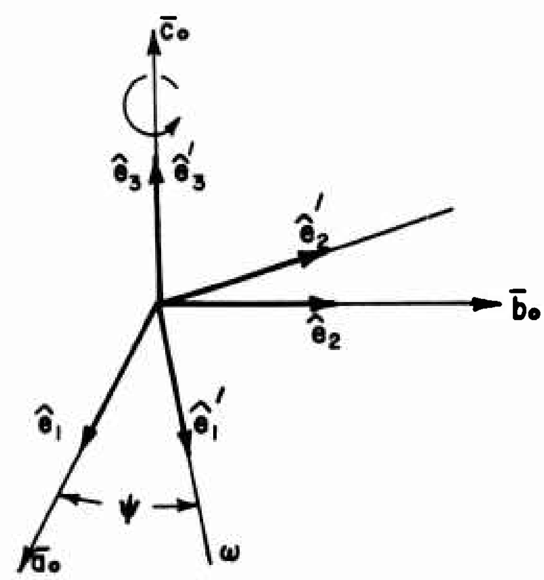

Fig. 9 Rotation about original normal

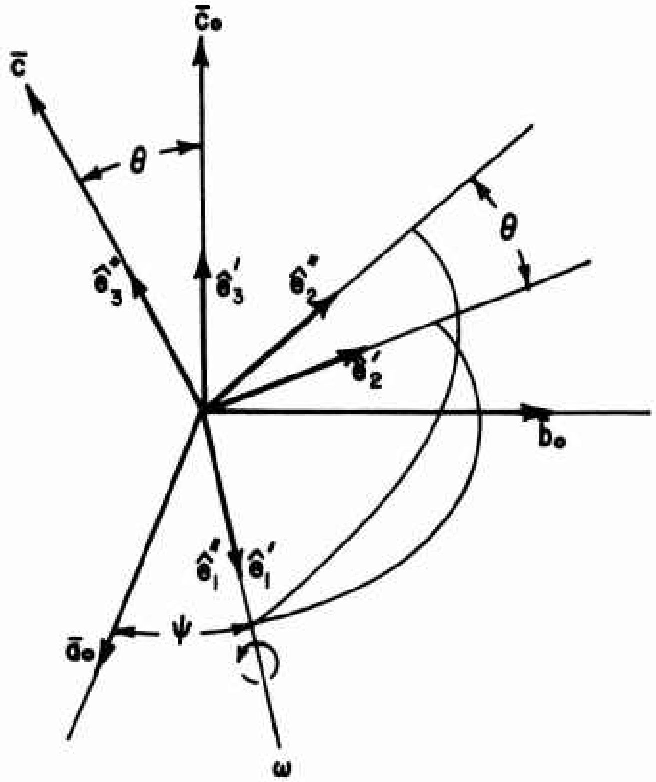

Fig. 10 Tilt

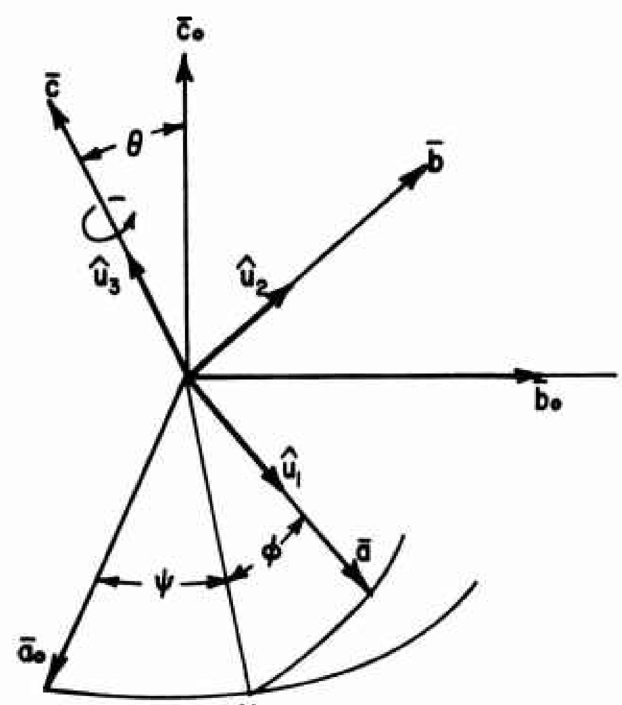

Fig. 11 Rotation about new normal

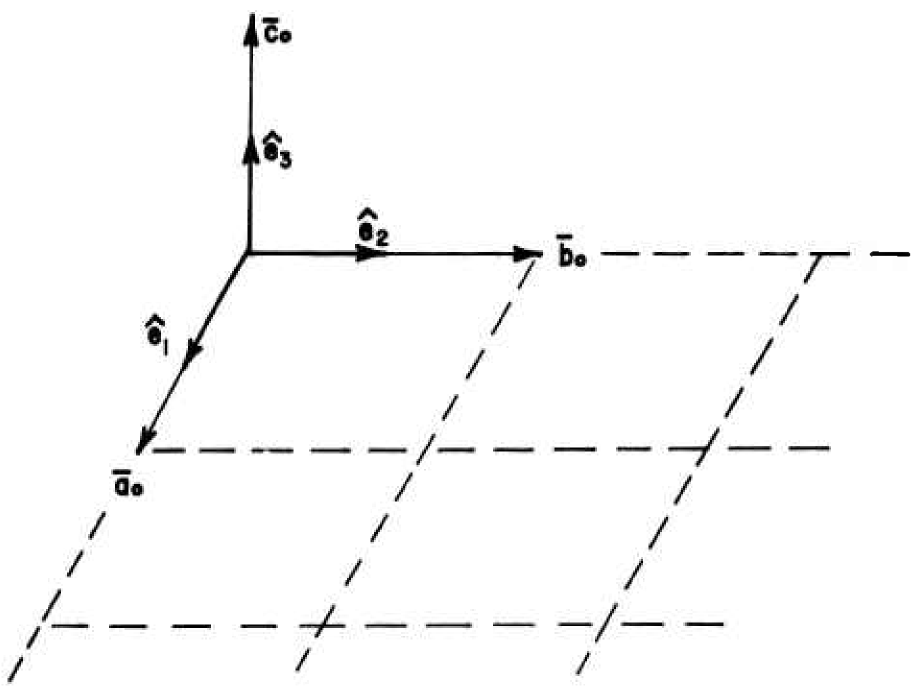

Fig. 12 Orthogonal lattice

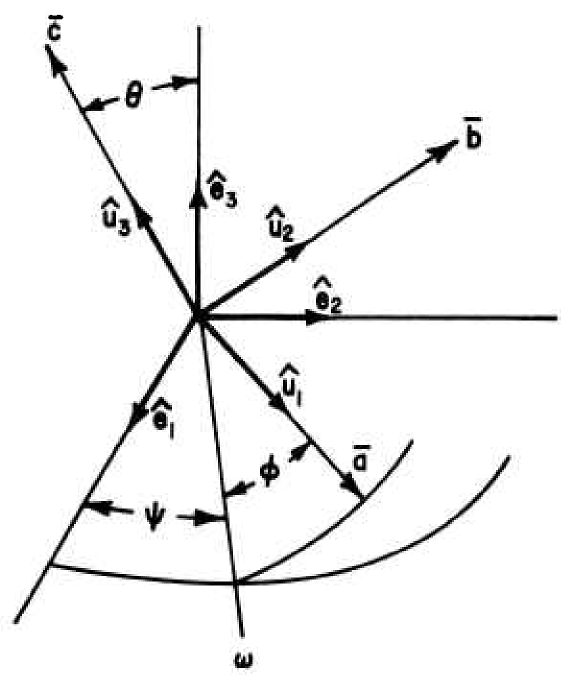

Fig. 13 Deformed lattice

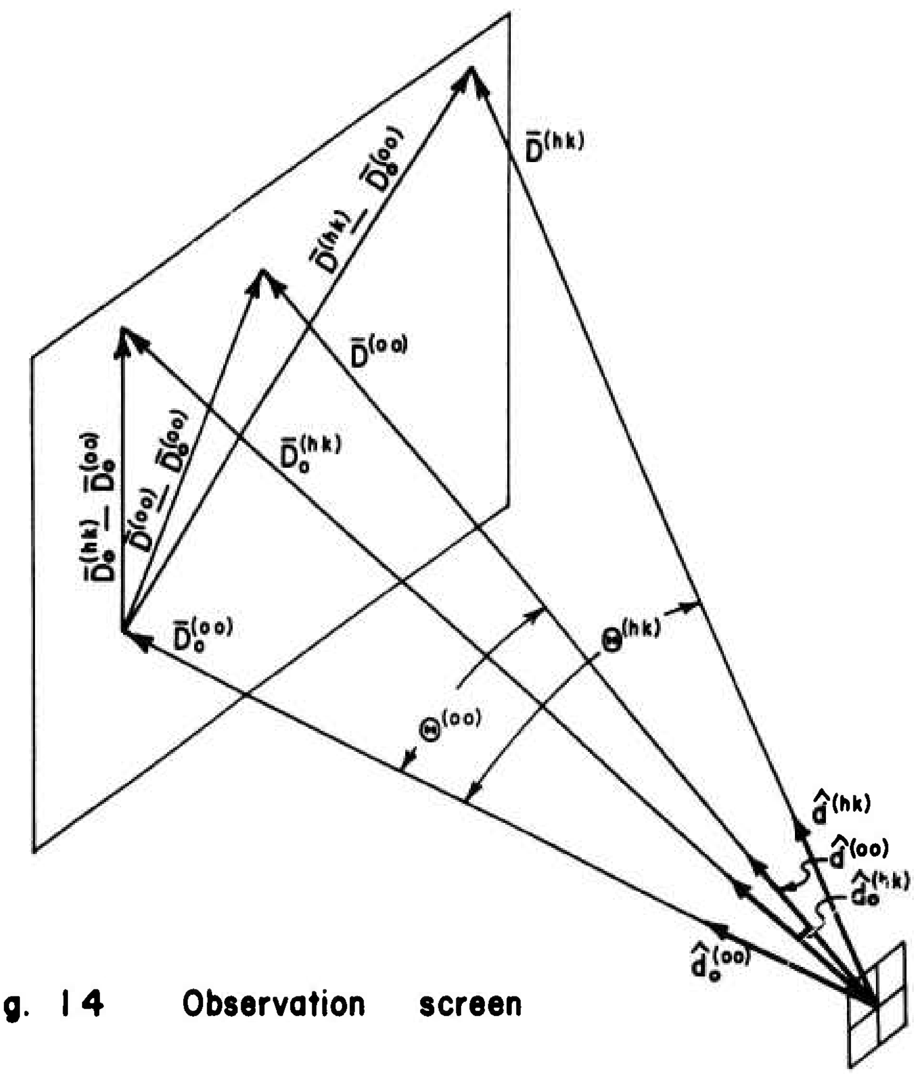

Fig. 14 Observation screen

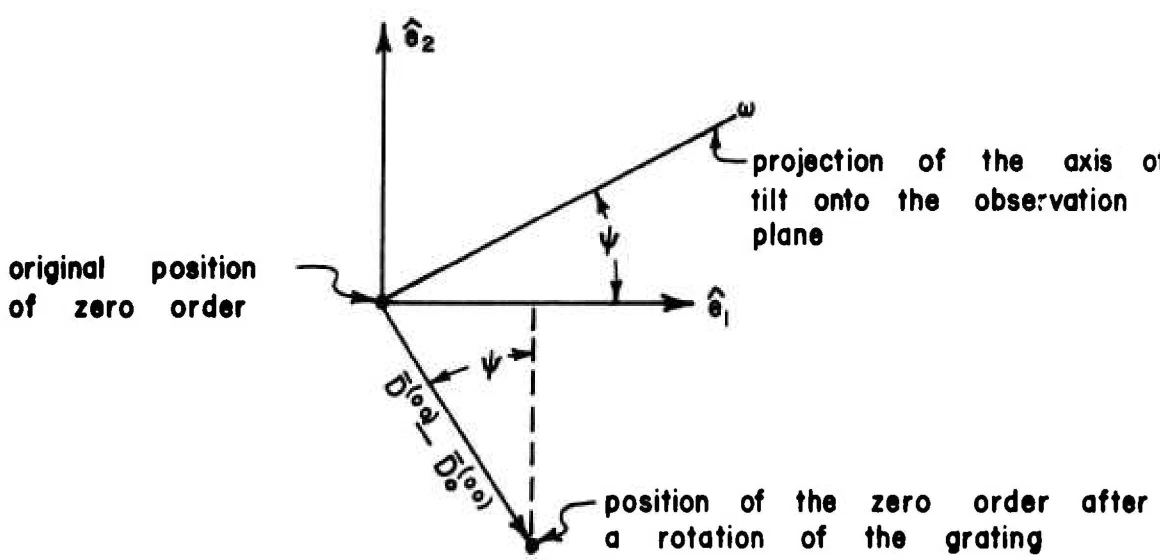

Fig. 15 Shift of zero order diffraction

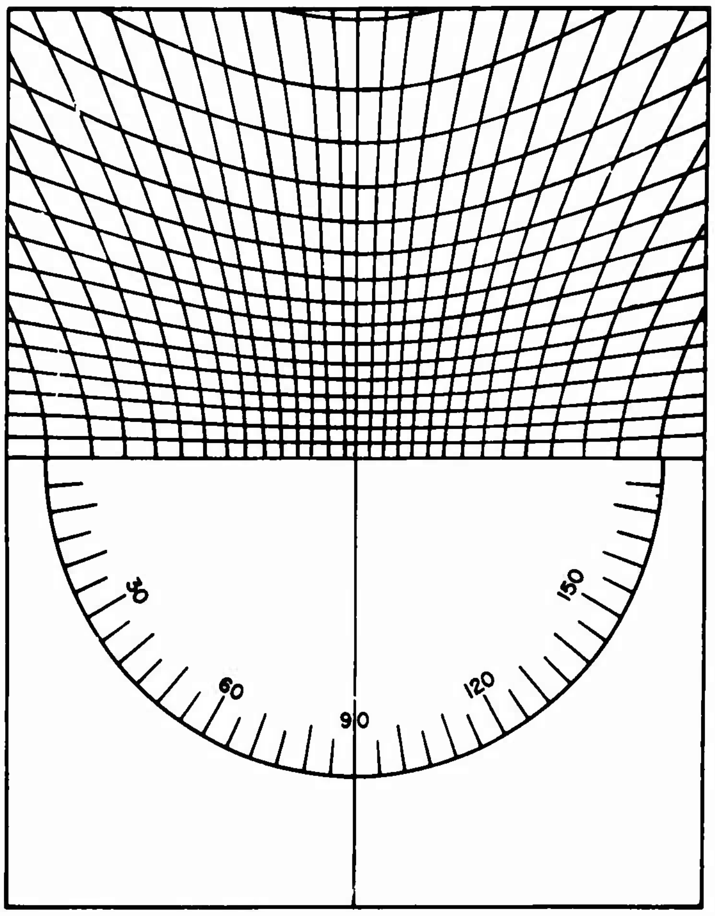

Fig. 16 Greninger chart for polychromatic back reflection X-ray technique

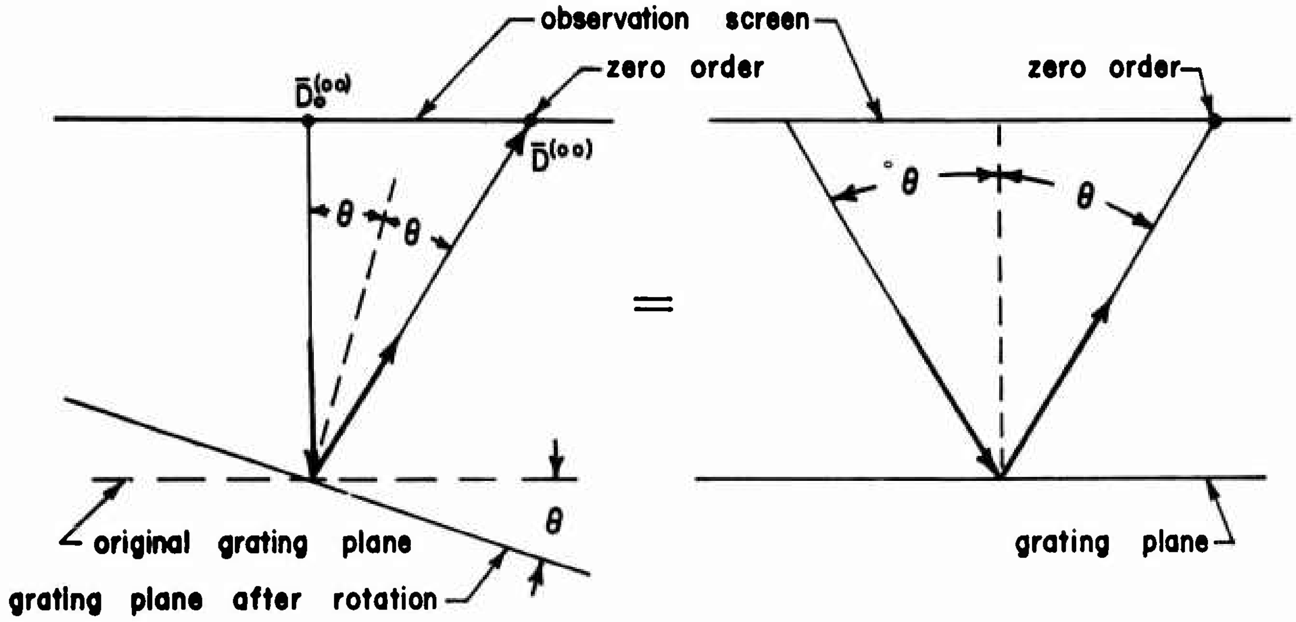

Fig. 17 Equivalent problems

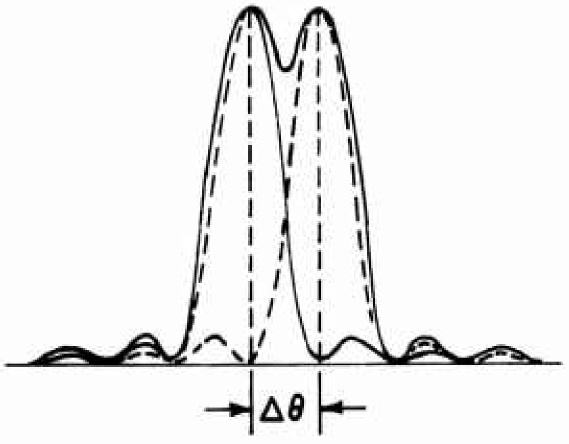

Fig. 18 Adjacent primary maxima

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13. ABSTRACT

General diffraction-strain equations are developed for the optical diffraction grating strain gages now coming into use in connection with experimental studies of wave propagation in solids. Equations are derived for the following cases: superimposed gratings crossed at any angle and subjected to arbitrary surface strains; orthogonal gratings subjected to arbitrary rotations; and orthogonal gratings aligned with principal strain directions and subjected to arbitrary rotations during strain.

The inherent strain resolution capability of an optical diffraction system is discussed and the differences between this technique for strain measurement and that of x-ray stress analysis are described.

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